Modernization of Agriculture and Long-Term Growth†

Dennis Tao Yang and Xiaodong Zhu

December 2008

Abstract

This paper develops a two-sector model that illuminates the role of agricultural modernization in the transition from stagnation to growth. When agriculture relies on traditional technology, industrial development reduces the relative price of industrial products, but has limited effect on per capita income because most labor must stay in farming. Growth is not sustainable until this relative price drops below a certain threshold, inducing farmers to adopt modern technology that uses industry-supplied inputs. Once agricultural modernization begins, per capita income breaks from stasis to modern growth. Our calibrated model accounts well for the experiences of the industrial revolution in England.

Keywords: long-term growth, transition mechanisms, relative price, agricultural modernization, structural transformation, industrial revolution, England

JEL classification: O41, O33, N13

†Contact information: Yang, Department of Economics, Virginia Tech and The Chinese University of Hong Kong, Email: deyang@cuhk.edu.hk; Zhu, Department of Economics, University of Toronto, Email: xzhu@chass.utoronto.ca.
“The man who farms as his forefathers did cannot produce much food no matter how rich the land or how hard he works. The farmer who has access to and knows how to use what science knows about soils, plants, animals, and machines can produce abundance of food though the land be poor. Nor need he work nearly so hard and long. He can produce so much that his brothers and some of his neighbors will move to town to earn their living.” — T. W. Schultz (1964)

1 Introduction

Sustained growth in living standards is a recent phenomenon. Estimates of GDP per capita for the world indicate dramatic differences in growth experience between earlier historical times and the last two centuries. Prior to 1820, the world economy was in a Malthusian state with little growth; per capita product in that year was only 50 percent higher than the level estimated for ancient Rome, according to Maddision (2001). Clark (2007) presents a similar view that the material lifestyle of the average person in the world around 1800 was about equivalent to that of the Stone Age. During the last two centuries, however, the world’s per capita output has increased eightfold. Because of its enormous welfare implications, understanding the switch from stasis to progress has become a central concern to economists interested in growth and development.

The existing literature on the transition to modern growth has focused mostly on the role played by human capital accumulation and technological change at the aggregate level. For instance, Becker, Murphy and Tamura (1990) and Lucas (2002) assign a central role to endogenous fertility choice and investment in human capital. An exogenous increase in the returns to training or education induces parents to choose fewer births but invest more in each child, thus leading to the takeoff in per capita income growth. Another line of ideas emphasizes the relationship between population growth and endogenous technological progress (e.g., Kremer 1993, Goodfriend and McDermott 1995, and Jones 2001). Higher population spurs technological advances, which eventually lead to the break from a stagnant equilibrium. By combining the above two strands of research, Galor and Weil (2000) consider the nexus between human capital investment and technological change as the key to transition. As technological change accelerates with population growth, human capital becomes more valuable in coping with changing economic conditions; therefore, the return to human capital increases and fertility declines. Hansen and Prescott (2002) suggest that what triggers modern growth is the adoption of a less land-intensive production technology that,
although available throughout history, was not previously profitable for individual firms to operate. The economy is trapped in the Malthusian regime when it only uses a traditional land-intensive technology that is subject to diminishing returns to labor.¹

While these papers provide powerful insights into the transition to modern growth, the existing literature on growth is largely silent about one important mechanism: the transformation of traditional agriculture. Admittedly, development economists have long emphasized the role of agriculture in economic growth.² Schultz (1964), in particular, argues that subsistence food requirement presents a fundamental challenge to poor economies and that modernization of agriculture is essential for sustained growth. This view is echoed by economic historians. For example, Wrigley (1990) states: “The economic law of diminishing marginal returns was inescapable. The future was therefore bound to appear gloomy as long as it seemed proper to assume that the productivity of the land conditioned prospects, not merely for the supply of food in particular, but also for economic growth generally. Only if there were radical and continuous technological advances in agricultural technology could this fate be avoided.”

Building on these insights from economic history and development literature, this paper develops and calibrates a two-sector model highlighting the importance of agricultural modernization as a central mechanism for the transition from stagnation to growth. Our model is motivated by three concurrent events that occurred in England between 1700 and 1909, a period encompassing the industrial revolution:

(a) the well-known fact that, around 1820, per capita GDP for the English economy ended a long flat trend, taking off to sustained growth (see Panel A of Figure 1);³

(b) the less well-known fact that the systematic adoption of farm machinery also began around 1820—the percentage of farms that owned agricultural machines was nearly nil in the beginning of the century, but the adoption became widespread in a few decades thereafter (e.g., Walton 1979; Overton 1996; see Figure 2)⁴; and

¹See Galor (2005) for a survey of the literature on stagnation to growth.
²Important contributions among the vast collection of the literature include Johnston and Mellor (1961), Jorgenson (1961), Schultz (1964) and Timmer (1988); Kelley, Williamson and Cheatham (1972) present an early numeric simulation of a two-sector model; Johnson (1997) provides a recent survey.
³Statistical information quoted in this paper are obtained from multiple sources. See section 5 and Appendix B for detailed data descriptions.
⁴For centuries in world development, the advancements in agricultural productivity were derived primarily
(c) perhaps the least known, but a central fact for our study, is that the relative price of industrial to agricultural products in England declined persistently for more than a century, hitting a low point in the 1820s, and then stabilized at that level in the following decades (see Panel B of Figure 1).

Are these observed patterns merely historical coincidences?

We argue in this paper that the three events are causally linked through intricate relationships between industrial and agricultural development. When agriculture relies on traditional technology, industrial development reduces the relative price of industrial to agricultural products, but has limited effect on per capita income, because most labor must stay in farming. Growth is not sustainable until this relative price drops below a certain threshold, making it profitable for some farmers to adopt modern technology that uses industry-supplied inputs. Industrial development is a necessary precondition for modernizing agriculture. Once agricultural modernization begins, per capita income breaks from stasis to growth in junction with coordinated movements in relative price, wage, land rent, and structural transformation. During the transition when modern technology is adopted by some but not all farmers, the relative price stabilizes to the threshold level such that farmers are indifferent towards using either of the two technologies.

More specifically, we model two sectors, agriculture and industry. Central to the analysis is the choice of two technologies potentially available to farmers: a traditional technology that uses labor and land, where the latter is in fixed supply, thus implying diminishing returns to labor. The alternative is a modern technology that also uses an intermediate input, which is produced by the industry. This input represents farm machinery in the paper, but could likewise refer to factors such as chemical fertilizers and high-yield seed varieties. The cost of the input is determined endogenously, depending in part on the industrial total factor productivity (TFP), which grows exogenously. Farmers start with the traditional technology. They begin to use the modern technology when the price of the intermediate input falls below a threshold level such that its adoption yields higher profits than only using the traditional

---

from the experiences of farm people. However, starting around 1820, in England and in other parts of the world such as the U.S., the application of scientific knowledge and inputs supplied by industry have become the engine of rapid agricultural productivity growth (Huffamn and Evenson, 1993; Johnson, 1997). This paper refers to agricultural modernization as the use of industry-supplied inputs in farming, which relates mainly to mechanization in the 19th century, but also including chemical, biological and other agronomic innovations in later periods.

Hence, industry corresponds to the rest of the economy other than agricultural production. We also use “nonagricultural sector” interchangeably with “industry”.
technology. Agricultural modernization is crucial in the model as it ignites the transition process.

In the traditional economy, slow TFP growth in experience-based farming systems requires a high employment share in agriculture to assure sufficient food supply. Positive shocks to agricultural productivity may lead to temporary structural transformation and per capita income increases. However, high income induces population growth, which in turn reduces per worker output in agriculture because of the fixed supply of land. TFP growth in industry can neither generate sustained structural transformation nor income growth, because most labor must stay in farming. Therefore, without modernizing agriculture, the economy cannot break away from the Malthusian trap.

In the long-run, however, industrial TFP growth lowers the relative price of industrial to agricultural products, which eventually leads to agricultural modernization. The transition to modern growth begins when the relative price drops below a critical level, inducing farmers to use the modern technology. During the transition, structural transformation accelerates and the economy steps on a path of sustained growth. The critical link is that, as industrial TFP grows, the cost of intermediate inputs declines and a larger quantity of intermediate inputs are used in agricultural production, hence raising agricultural labor productivity. In other words, with agricultural modernization, TFP growth in industry will join forces with TFP growth in agriculture, contributing directly to agricultural labor productivity growth through the use of intermediate inputs, thus facilitating structural change. In contrast to the traditional economy, where per capita income is constrained by agricultural TFP and population growth, TFP growth in both agriculture and industry contribute to per capita income growth. In the transition period, the relative price settles to a stable level such that farmers are indifferent to the two technologies. Continued industrial growth tends to lower relative price, but the effect is offset by the more widespread use of modern technology. The transition ends with the complete adoption of the new technology. Under modern growth, agriculture’s share of labor eventually approaches zero in the limit, while the growth rate of per capita income converges to the growth rate of industry.

Our model provides predictions about structural breaks and coordinated movements in several macroeconomic variables for the periods of stagnation, transition and modern growth. We calibrate the model and examine whether the model’s predictions are quantitatively consistent with the observations of the English economy between 1700 and 1909. From multiple data sources, we compile decennial time series of real GDP per capita, prices for agricultural and principal industrial products, agricultural mechanization, employment share
in agriculture, real average wages of adult farm workers, and land rent. We also rely on historical studies of the English economy to infer exogenous TFP growth in agricultural and nonagricultural production. We then calibrate the model to the English economy, generate the time paths for the six key aggregate economic variables, and present joint comparisons with their counterparts in the data. Our quantitative analysis accounts well for the observed English growth experiences during the industrial revolution. The findings, which are robust to incorporating food trade into the model, support a coherent view on the importance of agricultural modernization in making the transition from stagnation to growth possible.

Recently, burgeoning literature based on two-sector models has explored the role of structural transformation in growth.\(^6\) Stressing the importance of agriculture in a dual-economy model, our central idea lies in technological change within agriculture. This emphasis is closely related to Hansen and Prescott (2002), who study the growth implications of switching from traditional to modern technology in an aggregate model, and to Gollin, Parente and Rogerson (2007), who examine the effects of using alternative agricultural technologies on the evolution of international income differences.\(^7\) However, their papers have crucial differences from ours. Hansen and Prescott’s framework is essentially a one-sector model with two production technologies that produce a single good. Therefore, their model leaves no room to explore the implications of the subsistence food constraint, the interactions between industrial and agricultural development, and the relative price change as a key to agricultural modernization. Similar to us, however, Gollin, Parente and Rogerson emphasize the importance of modern agricultural technology on long-run growth; but, they take agricultural modernization as an exogenous event. In contrast, we endogenize the process of agricultural modernization based on farmers’ choice of technologies and further investigate the timing and mechanisms behind the transition process. Unique to our model is the emphasis on industrial development as a necessary precondition for agricultural modernization. Moreover, we calibrate our model to the English economy and show that the transition mechanisms we identify are quantitatively consistent with the growth experiences of England. As we do in this paper, Stokey (2001) also calibrates a model of the British industrial revolution for the period 1780-1850. However, her focus is on quantifying the contributions of growing


\(^7\)Restuccia, Yang and Zhu (2008) is a related paper that examines the role of barriers to using modern agricultural technology in accounting for cross-country income gaps.
foreign trade and TFP growth in individual sectors to overall growth rather than studying
the transition from stagnation to growth.

The rest of the paper is organized as follows. Section 2 presents the basic structures of
the two-sector model. In section 3, we analyze the equilibrium properties for a traditional
economy without the use of modern agricultural technology. Section 4 studies the features of
the transition to modern growth. In section 5, we document stylized patterns of the English
economy using data for the period 1700-1909 and present findings on how the predictions
of our calibrated model match with the main features of the British industrial revolution.
Section 6 presents concluding remarks.

2 The Two-Sector Model

A. Preferences and Endowments

Consider an economy in discrete time. There is a fixed amount of land $Z$ and $N_t$ identical
individuals in period $t$. Each individual owns $z_t = N_t^{-1}Z$ amount of the land and one unit of
time, which is supplied inelastically to work in the labor market. Let $w_t$ be the wage rate
and $r_t$ be the rental rate of land. Then, an individual’s income is

$$y_t = w_t + r_t z_t.$$  

There are two consumption goods, agricultural and nonagricultural (or industrial). Let
the agricultural good be the numeraire and $p_t$ be the price of the industrial good. Each
individual household consumes a constant amount $\bar{c}$ of agricultural good ($c_{at}$) and spends
the remaining income on the consumption of industrial good ($c_{nt}$). Therefore, we have

$$c_{at} = \bar{c},$$  
$$c_{nt} = p_t^{-1} (y_t - \bar{c}).$$

Each individual lives for one period and, at the end of period $t$, gives birth to $g_t$ children.
The land owned by the parent will be divided equally among the children. We assume that

---

8We prohibit trading in land ownership. Since households are identical in this economy, the assumption
is not substantial.
the population growth rate is a function of per capita income, \( g_t = g(y_t) \). Thus,

\[
N_{t+1} = g(y_t)N_t. 
\]

Since the agricultural good is used as the numeraire, the per capita income \( y_t \) is not the same as the usual measure of national income per capita, which is deflated by a GDP deflator. Rather, \( y_t \) is a measure of the household’s capacity to purchase agricultural goods. This corresponds well to the living standard measures used for low stages of development in the economic history literature, where they are often calculated as the ratio of nominal income to the price of commonly consumed food products.

**B. Production Technologies**

The nonagricultural good is produced with a linear production technology:

\[
Y_{nt} = A_{nt}L_{nt},
\]

where \( A_{nt} \) represents TFP in the industrial sector.

Two technologies are potentially available for farm production. The traditional technology only uses land and labor as inputs:

\[
Y^T_{at} = Z_1^{1-\sigma} (A_{at} L_{at})^\sigma, \quad 0 < \sigma < 1.
\]

Here, \( Z_t \) and \( L_{at} \) are land and labor inputs, respectively, where \( A_{at} \) denotes TFP in traditional agriculture\(^9\), \( \sigma \) is the labor share, and superscript \( T \) denotes traditional technology. The modern agricultural technology (with superscript \( M \)) uses an intermediate input \( X_t \), as well as the traditional inputs, land and labor:

\[
Y^M_{at} = \left[ Z_1^{1-\sigma} (A_{at} L_{at})^\sigma \right]^{1-\alpha} X_t^\alpha, \quad 0 < \alpha < 1.
\]

The intermediate input is produced outside of agriculture and has a factor share of \( \alpha \). The production of one unit of intermediate input requires \( \pi \) units of industrial output. Hence, the price of intermediate input is \( \pi p_t \). For simplicity, we assume that \( \pi = 1 \) for the rest of the paper.

Since the production technologies have constant returns to scale, we assume without

\(^9\)According to the production specification, the TFP in agriculture should be \( A^\sigma_{at} \) instead of \( A_{at} \). For exposition simplicity, however, we simply call \( A_{at} \) the agricultural TFP.
loss of generality that there is one stand-in firm in each of the two sectors. Both firms behave competitively, taking output and factor prices as given and choosing factor inputs to maximize profits. 

The stand-in firm (or farm) in agriculture has the following profit maximization problem:

$$\max_{Z_t^T, Z_t^M, L_t^T, L_t^M, X_t} \left\{ (Z_t^T)^{1-\sigma} (A_t L_t^T)^\sigma + \left[ (Z_t^M)^{1-\sigma} (A_t L_t^M)^\sigma \right]^{1-\alpha} X_t^\alpha \right\},$$

subject to the quantity constraints:

$$Z_t^T + Z_t^M = Z_t,$$

and

$$L_t^T + L_t^M = L_t.$$

The profit maximization problem of the industrial firm is

$$\max_{L_t} \{ p_t A_t L_t - w_t L_t \}$$

C. Technology Adoption in Agriculture

If the farm adopts the modern technology and allocates $Z_t^M (>0)$ amount of land and $L_t^M (>0)$ amount of labor to the production using modern technology, then, from (4), the optimal quantity of the intermediate input the farm uses is given by

$$X_t = \left( \frac{\alpha}{p_t} \right)^{1/(1-\alpha)} (Z_t^M)^{1-\sigma} (A_t L_t^M)^\sigma,$$

and the value-added produced by the modern agricultural technology is

$$\hat{Y}_{at}^M = Y_{at}^M - p_t X_t = (1 - \alpha) \left( \frac{\alpha}{p_t} \right)^{\alpha/(1-\alpha)} (Z_t^M)^{1-\sigma} (A_t L_t^M)^\sigma.$$

In comparison, if the farm uses the same amounts of land and labor for production using the traditional technology, the output is $(Z_t^M)^{1-\sigma} (A_t L_t^M)^\sigma$. Clearly, the farm will adopt modern technology only if

$$(1 - \alpha) \left( \frac{\alpha}{p_t} \right)^{\alpha/(1-\alpha)} \geq 1.$$ 

(6)

When the equality holds in (6), the farm is indifferent between the two technologies; either or both maybe used. Therefore, the farm would not adopt the modern agricultural technology if the cost of using the intermediate input $(p_t)$ is too high. To find the conditions for adopting
modern agricultural technology, we need to solve the equilibrium price $p_t$. Before doing that, we first define the competitive equilibrium in our model economy.

D. Market Equilibrium

**Definition 1** A competitive equilibrium consists of sequences of prices $\{w_t, r_t, p_t\}_{t \geq 0}$, firm allocations $\{L_t^T, L_t^M, L_{nt}, Z_t^T, Z_t^M, X_t\}_{t \geq 0}$, consumption allocations $\{c_{at}, c_{nt}\}_{t \geq 0}$, and the size of the population $\{N_t\}$, such that the following are true:

1. Given the sequence of prices, the firm allocations solve the profit maximization problems in (4) and (5).
2. Consumption allocations are given by (1) and (2).
3. All markets clear:

   \[ Y_{at} = N_t \tilde{c}_a, \]  
   \[ Y_{nt} = N_tC_{nt} + X_t, \]  
   \[ N_t = L_t^T + L_t^M + L_{nt}, \]  
   \[ Z_t = Z_t^T + Z_t^M. \]

4. Population growth rate is given by equation (3).

The following proposition holds for the competitive equilibrium:

**Proposition 1** Let $\Phi_l \equiv (1 - \alpha)^{\frac{\sigma - 1}{\sigma}} \alpha^{-1} c_{nt}^{\frac{\sigma - 1}{\sigma}}$ and $\Phi_h = (1 - \alpha)^{-\frac{1}{\sigma}} \Phi_l$. In agricultural production, the farm only uses traditional technology if

\[ \frac{A_{nt}}{A_{at} \left( \frac{Z}{N_t} \right)^{\frac{1-\sigma}{\sigma}}} \leq \Phi_l; \]

only uses modern technology if

\[ \frac{A_{nt}}{A_{at} \left( \frac{Z}{N_t} \right)^{\frac{1-\sigma}{\sigma}}} \geq \Phi_h; \]

and uses both technologies if

\[ \Phi_l < \frac{A_{nt}}{A_{at} \left( \frac{Z}{N_t} \right)^{\frac{1-\sigma}{\sigma}}} < \Phi_h. \]
Proof: The proofs of the propositions are provided in Appendix A.

This proposition identifies several factors that directly influence the use of modern agricultural technology. First, the TFP parameter \( A_{at} \) and land-to-population ratio \( Z/N_t \) are negatively related to the adoption of modern technology. Second, the industrial TFP (\( A_{nt} \)) has a positive effect on the adoption of modern farm technology. As we shall elaborate further, this is because high industrial productivity would lower the price of nonagricultural good, thus reducing the cost of using the industry-supplied intermediate input.

3 The Traditional Economy

We define a traditional economy as one in which farmers only use traditional technology. Proposition 1 suggests that if the initial land to population ratio \( Z/N_0 \) is sufficiently high and/or the initial relative TFP \( A_{a0}/A_{a0} \) is sufficiently low, the economy starts out as a traditional one. The following proposition states the determination of key variables in this economy.

Proposition 2 Let \( \tilde{A}_{at} = A_{at} (Z/N_t)^{1/\sigma} \), then, in the traditional economy, we have

\[
\begin{align*}
p_t &= \sigma \frac{\sigma - 1}{\sigma} \tilde{A}_{at}^{-1}, \\
w_t &= \sigma \tilde{A}_{at}, \\
r_t &= (1 - \sigma) \tilde{c} N_t / \tilde{Z}, \\
y_t &= \left[ 1 - \sigma + \sigma \tilde{c}^{\frac{1}{\sigma}} \tilde{A}_{at}^{-1} \right] \tilde{c}, \\
L_{at}/N_t &= \tilde{c}^{\frac{1}{\sigma}} \tilde{A}_{at}^{-1}. 
\end{align*}
\]

In period \( t \), both per capita income \( (y_t) \) and the employment share of agriculture \( (L_{at}/N_t) \) are determined by the variable \( \tilde{A}_{at} \), which can be interpreted as a measure of labor productivity in traditional agriculture that increases with agricultural TFP \( A_{at} \) and land-to-population ratio \( Z/N_t \). This is an intuitive result that higher agricultural TFP and land endowment imply higher agricultural labor productivity, which lead to higher per capita income and lower employment share in agriculture. Moreover, in period \( t \), rental price rises with population size; wage depends on agricultural labor productivity, and relative price \( (p_t) \) is determined by the relative productivity of agriculture and industry \( (\tilde{A}_{at}/A_{nt}) \).
The steady-state properties of the key variables can also be derived. Equations (14) and (15) suggest that a traditional economy can achieve sustained structural change (i.e., persistent decline in the employment share of agriculture) and per capita income growth only if there is sustained growth in $A_{at}$. By definition, we have

$$\frac{\tilde{A}_{at+1}}{A_{at}} = \frac{A_{at+1}}{A_{at}} \left( \frac{N_{t+1}}{N_t} \right)^{-\frac{\alpha}{\sigma}} = \frac{A_{at+1}}{A_{at}} \left[ g(y_t) \right]^{-\frac{1-\sigma}{\sigma}}$$

If $A_{at}$ grows at a constant rate $\gamma_a \geq 1$, then, the above equation becomes

$$\tilde{A}_{at+1} = \gamma_a \left[ g \left( (1 - \sigma + \sigma \bar{\tau}^{\frac{1}{\sigma}} \tilde{A}_{at}) \bar{\tau} \right) \right]^{-\frac{1-\sigma}{\sigma}} \tilde{A}_{at}.$$  \hspace{1cm} (16)

We make the following assumption about function $g(.)$.

**Assumption 1** (i) $g(\bar{\tau}) < 1$; (ii) there is a $\tilde{y} > \bar{\tau}$ such that $g(\tilde{y}) > \gamma_a^{\frac{\alpha}{\sigma}}$; and (iii) $g(.)$ is continuous and strictly increasing over the interval $[0, \tilde{y})$, decreasing over the interval $[\tilde{y}, \infty)$, and $\lim_{y \to \infty} g(y) = 1$.

Under this assumption, population growth rate increases with income when starting initially at a low income level. The population growth rate then increases to its peak at a certain income level, after which the growth rate declines with income and eventually converges to one. This hump-shaped function for population growth rate is consistent with typical patterns of demographic transition.

**Proposition 3** Under Assumption 1, there exists a unique steady state solution to the difference equation (16) such that the corresponding income per capita $y^* \in (\bar{\tau}, \tilde{y})$.

Therefore, without the adoption of modern agricultural technology, the economy always settles down at a Malthusian steady state with constant income per capita at $y^*$. There is no sustained growth in living standard. From equation (14), we know that the steady-state value of $\tilde{A}_{at}$, $\tilde{A}_{at}^*$, is determined by the equation

$$y^* = \left[ 1 - \sigma + \sigma \bar{\tau}^{\frac{1}{\sigma}} \tilde{A}_{at}^* \right] \bar{\tau}.$$ 

Since $y^* > \bar{\tau}$, $\tilde{A}_{at}^* > 0$. Thus, in the steady state, the population size is given by the equation:

$$\tilde{A}_{at}^* = A_{at} \left( \bar{z} / N_t \right)^{\frac{1-\sigma}{\sigma}}.$$ 

11
or

\[ N_t = \left( \frac{A_{nt}}{A_{nt}^*} \right)^{\frac{1}{\sigma}} Z. \]

Consequently, in the traditional economy, the effects of temporary agricultural TFP growth and any initial advantage in land endowment on agricultural labor productivity are completely offset by the adjustment of population size in the long run. As a result, labor productivity in agriculture is independent of agricultural TFP and land endowment. At the Malthusian steady state, as equations (11) to (15) show, per capita income, wage, and employment share of agriculture stay at constant levels; land rental rises with population; and relative price declines with industrial TFP growth.

4 Transition to Modern Growth

A. What Triggers Transition

We have shown that, without the use of modern agricultural technology, the economy is trapped in Malthusian stagnation. But will farmers eventually find it profitable to adopt the new technology?

Proposition 1 suggests that farmers will choose the modern intermediate input if

\[ \frac{A_{nt}}{A_{nt}^*} > \Phi. \]

Suppose the economy starts with a steady state equilibrium, where \( \tilde{A}_{nt} \) settles at a constant level \( \tilde{A}_{nt}^* \). Then, as long as \( A_{nt} \) grows without bounds, there will eventually be a time at which the inequality (17) will hold. The same point can be made from the behavior of the relative price of the nonagricultural good. From (11),

\[ p_t = \sigma c^{\frac{\sigma - 1}{\sigma}} \tilde{A}_{nt}/A_{nt}. \]

In the Malthusian steady state, we have

\[ p_t = \sigma c^{\frac{\sigma - 1}{\sigma}} \tilde{A}_{nt}^*/A_{nt}, \]

which declines monotonically with the growth of industrial TFP. Hence, at some point in time, the price of nonagricultural good will reach a low threshold level \( p^M = \alpha(1 - \alpha)^{1/\alpha} \) such that the adoption condition (6) holds with equality. At that point, farmers will start to use the intermediate input for agricultural production. Thus, continued industrial TFP growth, or persistent decline in the relative price, eventually triggers the transition from traditional agricultural technology to modern agricultural technology. Initially, when the relative productivity \( A_{nt}/\tilde{A}_{nt} \) just surpasses the threshold level \( \Phi_t \), but still below \( \Phi_h \), the
industrial TFP is not large enough to meet the demand for intermediate inputs by all farmers at a price that is profitable for them to use the modern technology. Under this situation, the economy is at an equilibrium by which some but not all farmers use the new technology and the relative price $p_t$ will stay at a level such that farmers are indifferent towards using either of the two technologies. We define the transition period—the time during which farmers use both technologies—a mixed economy.

**B. The Mixed Economy**

**Proposition 4** In the mixed economy,

$$p_t = p^M \equiv \alpha(1-\alpha)^{\frac{1-\alpha}{\sigma}},$$

$$w_t = p^M A_{nt},$$

$$r_t = (1 - \sigma) \left( \frac{\sigma \tilde{A}_{nt}}{p^M A_{nt}} \right) \frac{\sigma}{\tilde{N}_t \tilde{Z}},$$

$$y_t = p^M A_{nt} + (1 - \sigma) \left( \frac{\sigma \tilde{A}_{nt}}{p^M A_{nt}} \right) \frac{\sigma}{\tilde{N}_t \tilde{Z}},$$

$$\frac{L_{nt}}{N_t} = \left( \frac{\sigma \tilde{A}_{nt}}{p^M A_{nt}} \right)^{\frac{1}{1-\sigma}} \tilde{A}_{nt}^{-1},$$

$$\frac{Z_t^M}{Z} = \frac{1-\alpha}{\alpha} \left[ \left( \frac{A_{nt}}{A_{nt}} \Phi_l^{-1} \right)^{\frac{\sigma}{1-\sigma}} - 1 \right].$$

The time paths of these macroeconomic variables in the mixed economy differ significantly from the variables’ time paths in the traditional economy. More specifically, notice the structural breaks occurring in each of the variables:

- The relative price of industrial to agricultural products ($p_t$): In the traditional steady state, $p_t$ declines with the growth of $A_{nt}$ because $\tilde{A}_{nt}$ is a constant (see equation 11). Once agricultural modernization begins, $p_t$ settles to a constant level at which farmers are indifferent towards using either of the two technologies. Industrial TFP growth tends to lower relative price, but this would induce more widespread use of modern technology, which helps keep the relative price at the stable level.

- Per capita income ($y_t$): At the Malthusian equilibrium, per capita income $y_t$ is trapped at a low level because the slow growth of $A_{nt}$ is fully offset by population adjustment.
(see equation 14). During the transition, however, when the two sectors are integrated through the use of industry-supplied modern inputs, $A_{nt}$ contributes directly to per capita income, creating a clear structural break in the growth path of $y_t$. The modernization of agriculture helps the economy escape the Malthusian trap.

- The use of modern input in agriculture ($Z_t^{mt}/Z$): The ratio of land devoted to new technology over total land area measures the extent of modern technology adoption. In the agrarian economy, the old technology prevails. Once the transition begins, if the TFP in nonagriculture $A_{nt}$ grows sufficiently fast, $A_{nt}/\tilde{A}_{at}$ would increase over time, and the proportion of land (and labor) allocated to modern agricultural production would increase from zero to one, as $A_{nt}/\tilde{A}_{at}$ moves from $\Phi_l$ to $\Phi_h$ (see equation 24).

- Agriculture’s employment share ($L_{nt}/N_t$): In the traditional economy, the employment share is a decreasing function of $\tilde{A}_{at}$, which depends positively on $A_{nt}$ and $Z/N_t$ (see equation 15). Since $\tilde{A}_{at}$ tends to settle down at a steady state level, there could be no sustained structural change in the traditional economy. With mixed technologies, the share of employment in agriculture is also a decreasing function of $A_{nt}$, because TFP growth in industry reduces the cost of modern input $X_t$, thus inducing farmers to use more $X_t$ and less labor. Agricultural modernization makes sustained economic structural change possible.

- Wage rate ($w_t$): It is a constant at the Malthusian steady state. As the economy enters the transition, the wage rate grows with the industrial TFP $A_{nt}$.

- Land rent ($r_t$): During the transition, the land rental price is no longer a simple increasing function of the population size as in the traditional economy. The price of land is also affected by the relative TFP levels in the two sectors ($\tilde{A}_{at}/A_{nt}$) because the intermediate input has become a substitutable factor for land in agricultural production.

C. Modern Growth

When $A_{nt}$ grows sufficiently fast, the relative productivity $A_{nt}/\tilde{A}_{at}$ will continue to rise such that it will eventually reach the threshold $\Phi_h$. Thereafter, the economy enters the era of modern growth with complete adoption of the modern technology.
Proposition 5 Let $\tilde{A}_{at}^M = \left(\tilde{A}_{at}\right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} A_{nt}^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}}$. Then, in a modern economy, we have

\begin{align*}
p_t &= \sigma(1-\alpha) \left(\frac{\alpha}{\sigma(1-\alpha)}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \left(1-\frac{1}{\sigma(1-\alpha)}\right) \tilde{A}_{at}^M, \\
w_t &= \sigma(1-\alpha) \left(\frac{\alpha}{\sigma(1-\alpha)}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \left(1-\frac{1}{\sigma(1-\alpha)}\right) \tilde{A}_{at}^M, \\
r_t &= (1-\alpha)(1-\sigma)\tilde{A}_{at}, \\
y_t &= (1-\alpha) \left[1 - \sigma + \sigma \left(\frac{\alpha}{\sigma(1-\alpha)}\right)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \tilde{A}_{at}^M\right] \tilde{A}_{at}^M, \\
L_{at} = \frac{1}{N_t} &= \left(\frac{\sigma(1-\alpha)}{\alpha}\right)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \tilde{A}_{at}^{M-1}.
\end{align*}

In the modern economy, wage rate is a linear function of $\tilde{A}_{at}^M = \left(\tilde{A}_{at}^T\right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} A_{nt}^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}}$, which is a geometric average of the TFP levels in the two sectors. As such, TFP growth in both sectors contribute to the growth of per capita income. The growth rate of $\tilde{A}_{at}^M$ is given by

\begin{align*}
\frac{\tilde{A}_{at}^{M+1}}{\tilde{A}_{at}^M} &= \left(\frac{\tilde{A}_{at+1}^T}{\tilde{A}_{at}^T}\right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{A_{nt+1}}{A_{nt}}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \\
 &= \left(\frac{A_{at+1}}{A_{at}}\right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\frac{A_{nt+1}}{A_{nt}}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \left[g\left(y_t\right)\right]^{\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}.
\end{align*}

Suppose that $A_{at}$ and $A_{nt}$ grow at constant rates, $\gamma_a$ and $\gamma_n$. Then, the growth rate of $\tilde{A}_{at}^M$ becomes

\begin{align*}
\frac{\tilde{A}_{at}^{M+1}}{\tilde{A}_{at}^M} &= \left(\gamma_a\right)^{\frac{\sigma(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \left(\gamma_n\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \left[g\left(y_t\right)\right]^{\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}.
\end{align*}

Therefore, as long as $\left(\gamma_a\right)^{\sigma(1-\alpha)} \left(\gamma_n\right)^{\alpha} > \left[g\left(\bar{y}\right)\right]^{\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}$, $\tilde{A}_{at}^M$ will grow without bounds, and so will per capita income.

Summarizing all the results above, we have

Proposition 6 Under Assumption 1 and the assumption \(\left(\gamma_a\right)^{\sigma(1-\alpha)} \left(\gamma_n\right)^{\alpha} > \left[g\left(\bar{y}\right)\right]^{\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\), an economy that starts in a Malthusian steady state will at some point move into a mixed economy and eventually into a modern economy with sustained growth in per capita income.
During this process, the relative price of nonagricultural goods declines in the traditional economy, stays constant in the mixed economy, and then declines further in the modern economy. The employment share of agriculture starts to decline in the mixed economy and converges to zero in the modern economy. Land rent increases with population growth in both the traditional and the modern economy, and also depends on relative productivity growth during the transition period. Finally, real wage stays flat in the Malthusian regime, but begins to grow at the onset of the transition and indefinitely into the future.

5 Quantitative Analysis of the English Economy, 1700-1909

In this section we examine whether our calibrated model can quantitatively account for the growth experience in England for the period 1700 to 1909. We focus on long-term trends, structural breaks and coordinated movements across the six key macroeconomic variables—per capita GDP, relative price, agricultural mechanization, farm employment share, real wage of agricultural workers, and land rent. We first describe the data sources, and characterize major trends of the English economy, which will be followed by model calibration and discussions on our findings.

A. Data Compilation

Our quantitative analysis uses data of aggregate economic performance of the English economy for the period 1700 to 1909. The selection of the region is significant not only because the industrial revolution first occurred in England, but also because the availability of exceptionally rich historical data. We use England rather than the United Kingdom as the unit of analysis because data for Wales, Scotland, and Northern Ireland are incomplete for early historical periods. 1700 is chosen as the starting year of analysis as several data series—including by-sector employment shares and industrial output—are not available for earlier historical periods. The coverage ends at 1909, the year that concludes the first decade of the twentieth century, as the World War I opens another historical era. The period 1700-1909 is long enough to span across the essential stages of the transition from stagnation to growth during the British industrial revolution.

We construct data series on a decennial basis, emphasizing long-term trends with no attempt to account for short-term fluctuations. A decade consists of ten years starting with a rounded year of ten, i.e., 1700-1709; by this principle, 1909 marks the ending year of analysis.
Although data from 1910 to 1912 are available, we do not use the three-year data to represent decennial trends. The data series consists of constructed indices of real per capita GDP, population, employment share in agriculture, indices of agricultural mechanization, prices of agricultural products, prices of principle industrial products, real average day wages of adult farm workers, land rent, and food imports as percentage of domestic production. Moreover, we rely on historical studies of the English economy to obtain estimates for exogenous improvements in total factor productivity in both agricultural and nonagricultural production.

The data compilation is based on an extensive review of statistical sources as well as historical studies of the British economy. Completeness and reliability are two important criteria. Hence, our data are drawn heavily from two authoritative volumes of British historical statistics compiled by B. R. Mitchell (1962, 1988), who assembled best available data from government sources, censuses, historical studies, economists, statisticians and other independent scholarly publications. When certain data series are not available in Mitchell’s volumes, or cannot be traced back to 1700, we explore other historical studies. Among them, we have relied on the works of Clark (2001, 2002, 2004), Crafts and Harley (1992), Deane and Cole (1967), Wrigley and Schofield (1981), and other scholars. Appendix B describes in greater detail the sources and the construction of all key variables.

**B. The English Economy, 1700-1909**

Table 1 presents historical statistics of the English economy encompassing the entire process of the industrial revolution. In the period up to 1820, real per capita GDP fluctuated around a constant level, exhibiting typical features of a Malthusian regime. The employment share in agriculture declined gradually, which is consistent with slow increases in agricultural productivity. Starting in the early 1800s, the growth of per capita GDP and the pace of structural transformation began to accelerate. In the decades between 1820-9 and 1900-9, per capita GDP increased by a factor of 1.88 and the employment share of agriculture dropped from 33 percent to 10 percent. By 1909, England was far ahead of all other countries in the extent of structural transformation, and clearly England had already left behind the dreadful Malthusian regime.

The escape from Malthusian stagnation occurred concurrently with the adoption and diffusion of farm mechanization in England in the early 1800s. Despite the sparsity of historical data, John Walton creatively used farm sales advertisements to quantify the adoption of farm machines for selective regions in England and Wales for the years from 1753 to 1880 (see Walton 1979, Overton 1996, and details provided in Appendix B). Figure 2 reports the percentage of dispersal sales of farm stocks containing the sale of eight specific types of farm
machinery. The use of threshing, haymaking and chaff machines started around 1810, and the continuous adoption of turnip cutters began to appear around 1820. The diffusion of these machines, except for threshing machines, continued in an uptrend until 1880. Columns (6) and (7) of Table 1 present computed probabilities of adopting at least one and at least two agricultural machines on a farm during individual decades. For the case of two machines, the rate of possession by a typical farm was only 2 percent in 1810-9, but by 1880-9 the adoption rate had zoomed to 85 percent.

A central implication of the model is that the relative price of industrial to agricultural good falls continuously in the Malthusian steady state as a result of industrial TFP growth (see equation (18)). Then, during the transition to modern growth, the relative price should settle to a constant level, as equation (19) demonstrates. The observed English experience shows exactly this pattern (see Panel B of Figure 1). More specifically, as column (2) of Table 1 reveals, the relative price index declines rather persistently from 2.14 in 1700-9 to 1 in 1820-9, and then fluctuating around that level thereafter.

Table 1 also shows systematic patterns for the real wage of agricultural workers and land rent. The real wage stayed flat for more than a century, but began rising persistently after 1820. Throughout the period, real land rent exhibits an upward pattern, although the extent of the rise appears to be more pronounced in the first rather than the second period. Examining whether our model can account quantitatively for these coordinated movements in the key variables for the English economy is the task to which we now turn.

C. Model Calibration

For our quantitative exercise, each period in the model consists of 10 years, with the initial period starting in 1700-9. Initially, the model economy is in a Malthusian steady state, and in the 1820-9 period, the economy begins agricultural modernization, or the transition to modern growth. Technology parameters, subsistence consumption, initial TFP levels and population growth profile are calibrated. We then feed the TFP growth rates estimated from the historical data into the model to generate time series predictions of six key variables—per capita GDP, the relative price, agricultural mechanization, farm employment share, real wage of agricultural workers, and land rent, and compare them to their counterparts in the data. The details are as follows.

Technology parameters.—We set the labor share in traditional agriculture $\sigma$ to 0.6, consistent with Hansen and Prescott (2002) and Ngai (2004). Following Restuccia, Yang and Zhu (2008), we set the share of intermediate input in modern agriculture $\alpha$ to 0.4. The value of land endowment $\mathcal{Z}$ is normalized to one.
Initial values.—We normalize the initial value of income $y_0$ to one and set $N_0$ to the population level in England in 1700. From equation (14) and (15), the following equation holds in the traditional economy:

$$y_t = [1 - \sigma + \sigma \left( \frac{L_{at}}{N_t} \right)]\bar{c}.$$ 

Clark (2002) indicates that in 1700-9 the fraction of labor in agriculture is 0.55 in England. Hence, we can use the above equation to pin down the value of $\bar{c}$ such that the model implied initial income level $y_0$ is one. Given $N_0$ and the calibrated value of $\bar{c}$, we can use equation (15) to pin down the value of $A_{a0}$ such that the model implied value of $L_{a0}/N_0$ is 0.55. Because agricultural mechanization in England emerged in the early nineteenth century, or around 1820-9, to be more precise (Walton 1979; Overton, 1996), we choose the initial value of $A_{a0}$ such that in our model the use of modern agricultural technology would start in that decade.

Population growth profile.—We assume that population growth follows the same functional form as that in Hansen and Prescott (2002) and we use the observed England’s decennial population growth rates and per capita incomes to estimate the parameters of the function. Similar to their schedule, this estimated population growth function increases linearly at low income levels and then turns to decline at a slower rate through a linear scheme.

Total factor productivity growth.—For agriculture, Clark (2002) provides estimates for decennial TFP in English agriculture for the period 1500-1910 based on estimated factor prices, their input shares and output prices. He shows that agricultural TFP grew at a slow rate prior to 1860 and then increased rapidly from 1860 to 1910. Accordingly, we assume that $A_{at}$ grows at a decennial constant rate $\gamma_{a1}$ for the period 1700-1860 and grows at another constant rate $\gamma_{a2}$ for the period after 1860. Since agricultural TFP is $A_{a0}^{\sigma}$ in the model, the TFP growth rates in the two periods are $\gamma_{a1}^{\sigma}$ and $\gamma_{a2}^{\sigma}$, respectively. For each of the two periods, $i = 1, 2$, we regress $\log(TFP)$ on a time trend to obtain a slope coefficient $\xi_i$, which is the decennial exponential growth rate. Then, to obtain the decennial growth rate for $A_{at}$, we calculate $\gamma_{ai}$ according to the formula $\gamma_{ai} = \exp(\xi_i/\sigma)$.

With regards to TFP growth in nonagriculture, the pioneering work of Deane and Cole (1967) presents estimates of aggregate economic performance of the British economy for the period 1688-1959. However, as most economic historians agree, output growth during the industrial revolution was much slower than the view associated with the original estimates.
of Deane and Cole. To obtain an estimate for $A_{nt}$, we rely on the revised estimates of British industrial production by Crafts and Harley (1992) as a primary data source. Their results are widely accepted among economic historians, and have been used in recent quantitative studies of the British aggregate performance (e.g., Stokey, 2001).

We use estimates of output growth per worker of British industrial production to approximate exogenous improvements in TFP for the nonagricultural sector, i.e. $\frac{\Delta A_n}{A_n} = \frac{\Delta Y_n}{Y_n} - \frac{\Delta L_n}{L_n}$, an approach consistent with our model specification of linear production technology $Y_{nt} = A_{nt}L_{nt}$. More specifically, we first use the indices of British industrial production for the period 1700-1909, as covered in Crafts and Harley (1992), to compute the rate of industrial output growth ($\Delta Y_n / Y_n$). Crafts and Harley have estimated the annual growth rate of British industrial labor ($\Delta L_n / L_n$) at 0.8 percent for the period 1760-1801 and at 1.4 percent for the period 1801-1831; therefore, the decennial growth rate of $A_{nt}$ for the period from 1760 to 1831 can be inferred. For the period 1831-1909, we compute the decennial growth of the industrial labor force based on British population census as reported in Mitchell (1962). For the earlier period 1700-1760, Clark (2002) reports both share of adult male labor in agriculture ($s_a$) and estimates of male adult labor force in agriculture ($L_a$) by decade; therefore, we can compute the decennial non-agricultural labor force, i.e. $L_n = L_a(1 - s_a)/s_a$. By assuming that labor force in the non-agricultural sector grew at a similar rate as in the industrial sector, we can obtain nonagricultural TFP growth for the period 1700-1760.

D. Simulation Results

The primary objective of our model is to illuminate the transition mechanisms from stagnation to growth highlighting the causal linkages among several macroeconomic variables over the very long run. Although it is not our intention to provide a detailed model of the English growth experience, the success of the calibration certainly helps validate the model’s relevance. In this vein, we compare the predictions of the model on the six major variables to the data for the English economy over the period 1700-1909.

Figure 3 presents the time paths of the variables—actual data series vs. model predictions. The left middle panel contains information on the transition from traditional to mixed and modern economies. The line for model prediction denotes the fraction of productive inputs (land and labor) devoted to modern technology.\textsuperscript{10} In the traditional economy before

\textsuperscript{10}Since historical data on the percentage of land and labor allocated to modern technology is not readily available, the data measures used for the adoption of modern agricultural technology are estimated prob-
1810-9, farms only used the old technology, as the model implies. Agricultural mechanization begins in 1820-9, and the transition takes 7 periods (or decades), ending in 1880-9 with complete adoption of the new technology.

Overall, the time paths of the variables predicted by the model track well thestructural breaks and systematic trends that occurred in the English economy over more than two centuries. The simulation results support a coherent and unified view on the importance of agricultural modernization in making the transition from stagnation to growth. In the periods before 1820, the economy settles in the Malthusian steady state, where the per capita GDP and real wage both stay constant. The growth in industrial TFP leads to persistent decline in the relative price, but before reaching a low threshold level, farmers do not find it profitable to use modern productive inputs, resulting in no adoption of farm machinery. Agricultural productivity is low because of diminishing returns to labor due to the fixed supply of land. The model predicts no structural transformation during this period because low agricultural productivity limits the release of labor to industry.

When the relative price declines to the low critical level, profit-maximizing farmers will start to adopt the modern input produced by the industrial sector. Agricultural modernization triggers a virtuous cycle. As farmers substitute modern agricultural inputs for labor, structural transformation accelerates. As a result, per capita income emerges from stasis, breaking out to a high rate of growth; this is because once agricultural modernization begins, TFP growth in industry also begins contributing to aggregate growth [see equation (19)]. During the transition, the model’s predicted relative price settles to a constant, which is consistent with the data. By and large, the predicted wage and land rent also track the data well. While the rent displays an upward pattern throughout the period, the real wage stays flat for more than a century, but then rises persistently.

Despite the success of model simulations in matching the general time paths of the variables, there are two noticeable discrepancies. The first is the gap between the predicted relative price and the price revealed by data in the first century, although the downward

---

11 The model tracks the data well in the earlier period before 1820 as a larger population has a direct positive effect on land rent [see equation (14)]. During the transition, however, the determination of land rent becomes more complex, as equation (19) suggests.

12 This could be the result of under-estimating industrial TFP growth for the period 1700-1820, which in turn depends on historical estimates on industrial output, industrial labor force, and the assumptions regarding the growth rates of output and labor force across the industrial and the more inclusive nonagricultural
trends are very similar. The second discrepancy relates to the employment share of agriculture: while the observed decline is gradual and smooth, the predicted time path has a sharp kink in 1820—the trend is too flat initially, but then drops precipitously. Since structural transformation is important to our analysis, we expand our benchmark model below to see whether international trade would influence the change in agriculture’s employment share and the time paths of other major variables.

E. Incorporating Trade

England was a net food exporter in the first half of the 18th century, but turned into a net importer towards the end of the century (see Overton 1996; Deane and Cole 1967). Table 1 suggests that the initial growth in imports was gradual—the net food import relative to domestic production was merely 1 percent in 1780-9, and the ratio increased steadily to 17 percent in 1850-9. However, soon after the repeal of the English Corn Law, food imports exploded, finally reaching 76 percent of domestic production by 1900-9. The changes in food imports would clearly affect the pace of structural transformation and possibly the time paths of other macroeconomic variables for the English economy.

We incorporate food trade into the benchmark model, following the approach by Stokey (2001) who observes that England already imported significant amounts of food in the 1820s and exported roughly equal amounts of manufactured goods in terms of value added. We take food imports as exogenous and assume balanced trade such that the value of exports in nonagricultural goods is determined by the need to import food. Denote \( i_t \) as the percentage of food import relative to domestic food production for year \( t \). The market clearing conditions for the agricultural and nonagricultural goods become:

\[
(1 + i_t)Y_{at} = N_it, \tag{30}
\]

\[
Y_{nt} = N_tc_{nt} + X_t + E_t, \tag{31}
\]

where \( E_t \) is the amount of export in nonagricultural goods. We assume balanced trade so that

\[
\bar{p}_t w E_t = i_t Y_{at},
\]

where \( \bar{p}_t w \) is the relative price of the nonagricultural goods in the world market. Following Stokey (2001), we take \( \bar{p}_t w \) as an exogenous variable and assume it is at a level such that
trade is welfare enhancing for the domestic household at the margin. In Appendix A we show that this requires $p_t^w$ to be greater or equal to the relative price of nonagricultural goods under autarky. The solutions to this model with trade are identical to that of the benchmark model with one exception: the subsistence consumption requirement is changed to $\bar{c}/(1 + i_t)$.

Figure 4 presents model predicted time paths of all six key variables under scenarios with and without trade, and compare the simulation results with actual observations. By and large, the differences in predicted per capita GDP, relative price, real wage and land rent between the two scenarios are small. However, there is a noticeable discrepancy for the probability of adopting modern technology. When trade is taken into account, the model predicted fraction of agricultural inputs devoting to the new technology exhibits two periods of setbacks in 1860-9 and 1870-9, when food imports experienced dramatic growth. This pattern forms a contrast to the continuous diffusion of modern inputs under closed economy. The temporary reversal in technology adoption reflects reduced demand for domestic food production because of food imports, which in turn reduces the demand for the use of the intermediate inputs. However, after the short pause, the adoption continues to rise, similar to the case under autarky.

Figure 4 also reveals a significant impact of food imports on the pace of structural transformation. In contrast to an initial flat trend in agriculture’s employment share in the closed economy, the predicted employment share with trade exhibits gradual declines starting in the middle of the 18th century. Now the awkward kink around 1820 in the absence of trade disappears; the replaced line shows a much smoother decline in the fraction of labor in agriculture. After taking into account trade, the model can predict more accurately the extent of structural change, especially for the periods after 1850-9. Overall, the simulation results based on the model incorporating trade reinforce all the main findings of the benchmark model.

6 Concluding Remarks

History has witnessed persistent technological advances. Long before the industrial revolution, the Greeks and Romans discovered cement masonry, developed sophisticated hydraulic systems, and took great strides in advancing civil engineering and architecture. The

---

13 See Mokyr (1990) for a summary of major technological progress from the classical antiquities to the modern era of the later nineteenth century.
inventions arising in China, including paper, printing, the magnetic compass and gun powder, raised production efficiency through diverse channels. In the middle ages, dramatic improvements in energy utilization through the use of windmills, waterwheels, and horse technologies effectively expanded the production frontiers; and, the creation of the mechanical clock marked the entry of a key machine of the modern industrial age. Coming into the Renaissance period, along with remarkable scientific achievements, the innovations in shipbuilding, mining techniques, spinning wheels in textile production, and the use of blast furnaces raised the capacity of industrial production to new levels. Why did these major technological advances fail to generate sustained improvement in living standards?

We have argued in this paper that productivity growth in industry during early development was not enough to pull an economy out of a stagnant equilibrium. This is because low labor productivity associated with traditional agriculture requires much of the labor to produce food, thus imposing a constraint on per capita income growth. The decline in the relative price of industrial output not only reflects technological progress in industry, but acts as an agent—when falling below a critical level—inducing farmers to adopt a modern technology, which relies on industry-supplied inputs. Agricultural modernization ignites the transition to modern growth. Our analysis compliments the existing explanations for the transition to modern growth that focus on the role played by technological change and human capital accumulation. For instance, when structural transformation accelerates with modernizing agriculture, the rate of return to human capital is likely to rise because the dynamic environments of industries provide higher rewards for skill. Consequently, families would invest more in human capital and have fewer children. The average fertility rate would drop further because of a falling percentage of rural families. The emphasis on agricultural technology also provides specific content for long-term technological progress, allowing us to explore the timing and coordinated movements in macroeconomic variables through the transition from stagnation to growth.

Farm mechanization in England was only the beginning of agricultural modernization. In the past two centuries, the development of farm technology has been integrated into the rapidly expanding and increasingly complex systems of industrial and scientific advancements. The application of chemical and biological science has led to great inventions and has reduced the costs of fertilizers and new seeds, which have vastly improved agricultural productivity. In the United States, for instance, the labor used on farms to produce a ton of wheat or corn in the 1980s was about 1-2 percent of the labor used in 1800, and for a bale of cotton, only 1 percent (Johnson, 1997). In the twentieth century, labor productivity
growth in agriculture has generally outpaced the growth in other sectors of the economies in the industrial countries. The modernization of agriculture has been a crucial force behind sustained growth. In contrast, without much use of modern inputs, agricultural labor productivity in less developed countries is very low. As Restuccia, Yang and Zhu (2008) show, agricultural GDP per worker in the richest 5 percent of the countries in 1985 was 78 times that of the poorest 5 percent, while the difference in GDP per worker in non-agricultural sectors was only a factor of 5. Therefore, as our theory suggests, the provision and implementation of locally productive modern technologies in agriculture may contribute a great deal in helping the poorest countries escape from economic stagnation. Modernizing agriculture should be a central component in the design of any development policies.

Appendix A: Proofs of the Propositions

The proofs of Proposition 2, 4, and 5 are given in a not-for-publication appendix available from the authors by request. The proofs of Propositions 1 and 3 and the case of incorporating trade into the model are provided below.

Proof of Proposition 1

Let $\tilde{A}_{at} = A_{at}(\bar{z}/N_t)^{\frac{1-\sigma}{\sigma}}$. In a not-for-publication appendix we derive the equilibrium prices in the three possible cases as follows:

1. Traditional technology only:

$$p_t^T = \frac{\sigma}{\alpha c} \tilde{A}_{at}.$$

2. Modern technology only:

$$p_t^M = \sigma (1 - \alpha) \left( \frac{\alpha}{\sigma (1 - \alpha)} \right)^{\frac{\alpha}{\alpha + \sigma (1 - \alpha)}} \tilde{A}_{at}^M A_{nt}.$$

where

$$\tilde{A}_{at}^M = \left( \tilde{A}_{at}^T \right)^{\frac{\alpha (1 - \alpha)}{\alpha + \sigma (1 - \alpha)}} A_{nt}^{\frac{\alpha}{\alpha + \sigma (1 - \alpha)}}.$$

3. Both technologies are used:

$$p_t^{mixed} = \alpha (1 - \alpha)^{\frac{1-\alpha}{\alpha}}.$$
For given \( \tilde{A}_{at} \) and \( A_{nt} \), the economy uses traditional technology if and only if

\[
(1 - \alpha) \left( \frac{\alpha}{\bar{p}_t} \right)^{\frac{1}{1-\alpha}} \leq 1 \quad \text{and} \quad (1 - \alpha) \left( \frac{\alpha}{\bar{p}_M} \right)^{\frac{1}{1-\alpha}} < 1.
\]

This requires that

\[
\bar{p}_t^T = \sigma c^{\alpha(1-\sigma)/(\sigma(1-\alpha))} c^{(1-\sigma)(1-\alpha)} > \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}}.
\]

The later inequality is equivalent to the following:

\[
\bar{p}_t^T > \alpha(1 - \alpha)^{\frac{1-\sigma}{\alpha}} (1 - \alpha)^{\frac{1-\sigma}{\sigma}}.
\]

Apparently, as long as \( \bar{p}_t^T \geq \alpha(1 - \alpha)^{\frac{1-\sigma}{\alpha}} \), then the above inequality is automatically satisfied. Thus, the necessary and sufficient condition for the economy to use traditional technology is

\[
\sigma c^{\alpha(1-\sigma)/(\sigma(1-\alpha))} c^{(1-\sigma)(1-\alpha)} \leq \alpha(1 - \alpha)^{\frac{1-\sigma}{\alpha}}.
\]

For given \( \tilde{A}_{at} \) and \( A_{nt} \), the economy uses modern technology if and only if

\[
(1 - \alpha) \left( \frac{\alpha}{\bar{p}_t} \right)^{\frac{1}{1-\alpha}} > 1 \quad \text{and} \quad (1 - \alpha) \left( \frac{\alpha}{\bar{p}_M} \right)^{\frac{1}{1-\alpha}} \geq 1.
\]

Again, the two conditions can be written as

\[
\bar{p}_t^M = \sigma c^{\alpha(1-\sigma)/(\sigma(1-\alpha))} c^{(1-\sigma)(1-\alpha)} \leq \alpha(1 - \alpha)^{\frac{1-\sigma}{\alpha}}.
\]

Both will be satisfied if the later is satisfied. Hence, the necessary and sufficient condition for the economy to use modern technology only is

\[
\sigma c^{\alpha(1-\sigma)/(\sigma(1-\alpha))} c^{(1-\sigma)(1-\alpha)} \leq \alpha(1 - \alpha)^{\frac{1-\sigma}{\alpha}}.
\]
or
\[
\frac{A_{nt}}{A_t (Z/N_t)^{\frac{1-\sigma}{\sigma}}} \geq \Phi_h.
\]
When
\[
\Phi_l < \frac{A_{nt}}{A_t (Z/N_t)^{\frac{1-\sigma}{\sigma}}} < \Phi_h,
\]
neither traditional technology only nor modern technology only can be an equilibrium. The only possible equilibrium is when both technologies are used.

**Proof of Proposition 3**

In the steady state, equation (16) becomes
\[
1 = \gamma_a [g(y^*)]^{-\frac{1-\sigma}{\sigma}}
\]
or
\[
g(y^*) = \gamma_a^{\frac{\sigma}{1-\sigma}}.
\]
Under Assumption 1, \( g \) is continuous and strictly increasing over the interval \([\bar{c}, \bar{y}]\) and \( g(\bar{c}) < 1 \leq \gamma_a^{\frac{\sigma}{1-\sigma}} < g(\bar{y}) \). Therefore, there exists a unique \( y^* \in (\bar{c}, \bar{y}) \) such that \( g(y^*) = \gamma_a^{\frac{\sigma}{1-\sigma}} \).

The corresponding \( \tilde{A}_{a, t}^* \) is given by
\[
y^* = \left[ 1 - \sigma + \sigma \bar{c}^{\frac{1}{\sigma} \tilde{A}_{a, t}^*} \right] \bar{c}
\]
or
\[
\tilde{A}_{a, t}^* = \left[ \frac{y^*}{\bar{c}} - 1 + \sigma \right] \sigma^{-1} \bar{c}^\frac{1}{\sigma} > 0.
\]

**Incorporating Trade**

With trade, all the equilibrium conditions are the same as before except for the following two market clearing conditions for the agricultural and non-agricultural goods:

\[
(1 + i_t)Y_{at} = N_t \bar{c}, \quad (32)
\]
\[
Y_{nt} = N_t c_{nt} + X_t + E_t. \quad (33)
\]

Here \( E_t \) is the amount of export in nonagricultural goods. Since the market clearing condition (32) can be rewritten as
\[
Y_{at} = N_t \bar{c}_{t},
\]
where $\tau_t = \tau / (1 + i_t)$. Propositions 1 to 5 stay the same if we replace $\tau$ with $\tau_t$. Note that the condition for balanced trade requires that

$$p_t^w E_t = i_t Y_{at},$$

or

$$E_t = p_t^{w-1} i_t Y_{at} = p_t^{w-1} N_t \frac{i_t}{1 + i_t} \tau.$$

Substituting it into (33) yields the following:

$$c_{nt} = \frac{Y_{nt} - X_t}{N_t} - p_t^{w-1} \frac{i_t}{1 + i_t} \tau.$$

If the economy is in the traditional regime, we have $X_t = 0$ and

$$\frac{Y_{nt}}{N_t} = A_{nt} \left( 1 - \frac{L_{nt}}{N_t} \right) = A_{nt} \left( 1 - \frac{\tau_t}{A_{at}} \right).$$

Thus,

$$c_{nt} = A_{nt} \left( 1 - \frac{\tau_t}{A_{at}} \right) - p_t^{w-1} \frac{i_t}{1 + i_t} \tau.$$

Since per capita consumption of the agricultural good is always $\tau$, the representative household’s welfare is determined by the consumption of the nonagricultural good $c_n$. For the trade to be welfare non-decreasing at the margin, we need to have the following condition:

$$\frac{\partial c_{nt}}{\partial i_t} \geq 0.$$

From the equation for $c_{nt}$ above, we have

$$\frac{\partial c_{nt}}{\partial i_t} = \frac{1}{\sigma} \tau_t^{\frac{1}{\sigma}} - A_{nt} \frac{1}{A_{at}} \frac{1}{(1 + i_t)^2} - p_t^{w-1} \frac{1}{(1 + i_t)^2} \tau.$$

Thus, $\partial c_{nt}/\partial i_t \geq 0$ is equivalent to

$$\frac{1}{\sigma} \tau_t^{\frac{1}{\sigma}} - A_{nt} \frac{1}{A_{at}} \frac{1}{(1 + i_t)^2} - p_t^{w-1} \frac{1}{(1 + i_t)^2} \tau \geq 0$$

or

$$p_t^w \geq \sigma \tau_t^{\frac{1}{\sigma}} \frac{A_{nt}}{A_{at}} = p_t^T.$$

That is, the condition for trade being welfare improving requires the relative price of the
nonagricultural good in the world market to be higher than or equal to the domestic relative price. Similar conditions can be proved for the cases of the mixed economy and modern growth.

Appendix B: Description of Data

Population and sectoral labor share.—The first population census of Great Britain was conducted in 1801 and once every ten years thereafter. We use the arithmetic average of the 1801 and 1811 figures in England as its population for the decade 1800-09, and apply the same estimate for later decades. For the period 1700 to 1799, we deploy the yearly population estimates by Wrigley and Schofield (1981), which are reconstructed from local parish registers. To be consistent with the timing of the census, a simple arithmetic average of yearly population figures—starting from the first year of a decade to the first year of the following decade— is used as the decennial population figure. Then, we connect population figures from the two sources to cover the entire period 1700-1909.

The share of employment in agriculture is approximated by the share of males employed in agriculture (Clark, 2001), where the number of farm workers are estimated from population censuses from 1801 on. For the years before 1800, Clark builds on an estimate made by Lindert and Williamson (1982) that the farm labor force was no more than 53 percent of the male adult population in the 1750s. Clark applies a linear interpolation method to recover the shares of male labor in agriculture for the period 1750-1800 and applies an income elasticity approach to recover the employment share in agriculture back to 1700.

Per capita GDP.—For the period 1700 to 1869, Clark (2001) provides decennial real GDP per capita for England and Wales. We use this data series for England with the implicit assumption that GDP per capita is the same across these two regions, an assumption that is often made by economic historians in similar constructions of income data. To construct real GDP per capita for the period 1870-1909, we use the growth rates of real GDP per worker reported in the latest study by Feinstein (1990). We use his updated figures because Feinstein’s earlier estimates on GDP per capita were regarded as best available information (Mitchell, 1988). Based on Feinstein (1990), the growth rates of real GDP per worker in the UK was 1.32 percent for the period 1856-73, 0.9 percent for 1873-82, 1.43 percent for 1882-99, and 0.31 percent for 1899-1913. Using these figures, we compute weight-adjusted decennial growth rates for real GDP per worker for the four decades from 1870 to 1909, and therefore we are able to form an index of decennial real GDP per worker. Combining with information on the shares of labor force in the total population from decennial population
census (Mitchell, 1962), we are able to construct an index of decennial real GDP per capita for the UK. Connecting this index with the index for earlier decades reported in Clark gives an index of real GDP per capita for England for the period 1700-1909.

**Agricultural mechanization and food import.**—The systematic adoption and diffusion of farm machinery in England began in the early 1800s. Despite the sparsity of historical data, John Walton creatively relied on farm sales advertisements to quantitatively document the adoption of farm machines for selective regions in England and Wales for the period between 1753 and 1880 (Walton 1979). The original data consist of 3,115 advertisements of dispersal sales of farm stocks that appeared in the *Reading Mercury* and *Jackson’s Oxford Journal* in Oxfordshire in England. Walton’s study presents time series information on the percentage of farm households adopting each of the eight farm machines, which include turnip cutters, cake crushers, and reaping, mowing, haymaking, chaff, threshing and winnowing machines.

We construct two decennial indices of agricultural mechanization for England for the period 1700-1909. Using information on farm ownership of specific machines, we compute the first index as the probability of adopting at least one machine for a typical farm household and the second index as the probability of adopting at least two machines during individual decades. We use these two indices to approximate the extent of agricultural mechanization.

The quantitative analysis of the model, which assumes subsistence food consumption in a closed economy, requires making adjustments to English food trade with other economies. In particular, the simulation of the model uses information on food import or export as percentage of domestic agricultural production. While Mitchell (1962) reports the value of net food imports for the United Kingdom from 1854 onwards, estimates of earlier years have to draw from other sources. Overton (1996) and Deane and Cole (1967) both present decennial data of food imports relative to domestic production, but there are trade-offs in choosing among the two sources – the former covers a longer data series from 1700 to 1859 but with missing values for several decades, while the later has a shorter series from 1700 to 1820 without missing values. On the whole the two data series report very consistent trends. We construct a net import/output series for the period 1700-1851 based on the Overton series by applying a linear interpolation scheme to fill in the missing data. For the period 1850-1909, we divide the value of decennial imports (grain and flour plus meat and animals) from Mitchell (1962, pp.298-300) by the value of agricultural production also reported by Mitchell (1962, p.366). We connect the two time series by normalizing the overlapping decade of 1850 to a common value.

**Relative price.**—Clark (2004) uses a consistent method to construct an annual price series
for English agricultural output in the years 1209-1912. The series consists of information from 26 commodities: wheat, barley, oats, rye, peas, beans, potatoes, hops, straw, mustard seed, saffron, hay, beef, mutton, pork, bacon, tallow, eggs, milk, cheese, butter, wool, firewood, timber, cider, and honey. We take the arithmetic average of farm price indices within decades to form our decennial agricultural price series for the period from 1700 to 1909.

There is no single data source that provides aggregate price series on nonagricultural production for the English economy during our covered historical period. However, the work of Mitchell (1962) contains sufficient information that enables the construction of a long price series for principle industrial products. For the period between 1700 and 1800, we use the Schumpeter-Gilboy price indices for producer’s goods, which consist of 12 industrial products—bricks, coal, lead, pantiles, hemp, leather backs, train oil, tallow, lime, glue, and copper. This series ends in 1801.

To continue the price series for the period 1800-1913, we adopt the Rousseaux price index for principal industrial products, which has significant overlapping of product coverage with the Schumpeter-Gilboy price index (Mitchell, 1962). From 1800 to 1850, the Rousseaux price index covers coal, pig iron, mercury, tin, lead, copper, hemp, cotton, wool, flax, tar, tobacco, hides, skins, tallow, hair, silk, and building wood. For the years between 1850 and 1909, the index covers coal, pig iron, tin, lead, copper, hemp, cotton, wool, linseed oil, palm oil, flax, tar, jute, tobacco, hides, skins, foreign tallow, native tallow, silk, and building wood. We connect the two price indices and use it as constructed price series for the nonagricultural sector.

Wage and land rent.—In his study of agricultural performance and the industrial revolution, Clark (2002) assembles data from various published sources of key variables for English agriculture, 1500-1912. Based on Clark’s analysis, we use the average day wages of adult male farm workers outside harvest as a proxy for the wages of basic labor—the trend of this series closely resembles the changes in real wages for all workers from 1770 to 1870, a period studied by Feinstein (1998). The land rents are the market rental values of farmland including payments for tithe and taxes.
References


<table>
<thead>
<tr>
<th>Year</th>
<th>Real per capita GDP (1)</th>
<th>Relative price (Pn/Pa) (2)</th>
<th>Employment share in agriculture (3)</th>
<th>Real wage agricultural labor (4)</th>
<th>Real land rent (5)</th>
<th>Farm machines n≥1 (6)</th>
<th>Farm machines n≥2 (7)</th>
<th>Food imports as fraction of domestic production (%) (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700-9</td>
<td>0.80</td>
<td>2.14</td>
<td>0.55</td>
<td>1.01</td>
<td>0.62</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.02</td>
</tr>
<tr>
<td>1710-9</td>
<td>0.79</td>
<td>1.89</td>
<td>0.54</td>
<td>0.95</td>
<td>0.64</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.03</td>
</tr>
<tr>
<td>1720-9</td>
<td>0.83</td>
<td>1.83</td>
<td>0.53</td>
<td>0.97</td>
<td>0.69</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.04</td>
</tr>
<tr>
<td>1730-9</td>
<td>0.93</td>
<td>1.91</td>
<td>0.52</td>
<td>1.13</td>
<td>0.73</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.05</td>
</tr>
<tr>
<td>1740-9</td>
<td>0.85</td>
<td>1.95</td>
<td>0.52</td>
<td>1.10</td>
<td>0.67</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.07</td>
</tr>
<tr>
<td>1750-9</td>
<td>0.86</td>
<td>1.77</td>
<td>0.53</td>
<td>1.01</td>
<td>0.77</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.06</td>
</tr>
<tr>
<td>1760-9</td>
<td>0.84</td>
<td>1.83</td>
<td>0.49</td>
<td>0.98</td>
<td>0.75</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.03</td>
</tr>
<tr>
<td>1770-9</td>
<td>0.85</td>
<td>1.59</td>
<td>0.47</td>
<td>0.92</td>
<td>0.75</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>1780-9</td>
<td>0.82</td>
<td>1.71</td>
<td>0.44</td>
<td>0.97</td>
<td>0.74</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>1790-9</td>
<td>0.82</td>
<td>1.54</td>
<td>0.40</td>
<td>0.89</td>
<td>0.75</td>
<td>0.04</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>1800-9</td>
<td>0.84</td>
<td>1.29</td>
<td>0.37</td>
<td>0.81</td>
<td>0.75</td>
<td>0.07</td>
<td>0.00</td>
<td>0.06</td>
</tr>
<tr>
<td>1810-9</td>
<td>0.91</td>
<td>1.13</td>
<td>0.35</td>
<td>0.87</td>
<td>0.87</td>
<td>0.23</td>
<td>0.02</td>
<td>0.09</td>
</tr>
<tr>
<td>1820-9</td>
<td>1.00</td>
<td>1.00</td>
<td>0.33</td>
<td>1.00</td>
<td>1.00</td>
<td>0.42</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>1830-9</td>
<td>1.02</td>
<td>0.97</td>
<td>0.30</td>
<td>1.05</td>
<td>1.01</td>
<td>0.49</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>1840-9</td>
<td>1.06</td>
<td>0.96</td>
<td>0.26</td>
<td>1.11</td>
<td>1.08</td>
<td>0.75</td>
<td>0.29</td>
<td>0.15</td>
</tr>
<tr>
<td>1850-9</td>
<td>1.07</td>
<td>1.08</td>
<td>0.24</td>
<td>1.18</td>
<td>1.08</td>
<td>0.85</td>
<td>0.48</td>
<td>0.17</td>
</tr>
<tr>
<td>1860-9</td>
<td>1.08</td>
<td>1.03</td>
<td>0.21</td>
<td>1.18</td>
<td>1.11</td>
<td>0.91</td>
<td>0.62</td>
<td>0.24</td>
</tr>
<tr>
<td>1870-9</td>
<td>1.23</td>
<td>0.94</td>
<td>0.17</td>
<td>1.44</td>
<td>1.19</td>
<td>0.95</td>
<td>0.77</td>
<td>0.46</td>
</tr>
<tr>
<td>1880-9</td>
<td>1.40</td>
<td>0.90</td>
<td>0.15</td>
<td>1.69</td>
<td>1.24</td>
<td>0.98</td>
<td>0.85</td>
<td>0.56</td>
</tr>
<tr>
<td>1890-9</td>
<td>1.61</td>
<td>0.91</td>
<td>0.12</td>
<td>2.02</td>
<td>1.26</td>
<td>N.A.</td>
<td>N.A.</td>
<td>0.66</td>
</tr>
<tr>
<td>1900-9</td>
<td>1.88</td>
<td>1.04</td>
<td>0.10</td>
<td>2.08</td>
<td>1.13</td>
<td>N.A.</td>
<td>N.A.</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Note: the figures in columns (1), (2), (4) and (5) are indices of the corresponding variables with their 1820-09 values normalized to one. Columns (6) and (7) report computed probabilities of adopting at least one and at least two agricultural machines on a farm during individual decades. See Appendix B for details of constructing these time series.
Figure 1. Real Per Capita GDP and the Relative Price of Industrial to Agricultural Products in England, 1700-1909

A. Real per capita GDP

B. Relative price
Figure 2. Percentage of Farms Adopted Agricultural Machinery for Oxfordshire Regions in England: 5-year Moving Means

Figure 3. Trends of Major Macroeconomic Variables in England, 1700-1909

Note: Data 1 and 2 in the left middle panel are computed probabilities of adopting at least one and at least two agricultural machines on a farm in individual decades.
Figure 4. Trends of Major Macroeconomic Variables in England:
Simulations with and without Trade, 1700-1909

Note: Data 1 and 2 in the left middle panel are computed probabilities of adopting at least one and at least two agricultural machines on a farm in individual decades.