Can’t SBTC Explain the U.S. Wage Inequality Dynamics?

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Abstract

Based on the effect of skill-biased technology change (SBTC), this paper builds an undirected search model with heterogeneous firms and workers to explain the dynamics of the wage inequality in the U.S. from 1963-2005. Firms differ in capital intensity (technology content) of the job created and workers differ in their education level. As the match-specific productivity is stochastic, the productivity threshold of employment of each education-job pair matched is endogenously determined. The advance in skill-biased technology increases the productivity of the highly educated workers as well as the capital intensity facing firms when creating high-tech jobs. We argue that in response to the rise in the capital intensity, high-tech firms increase the productivity thresholds of hiring, which leads to wage increment in the high-tech sector, and thus widening the residual wage inequality. Meanwhile, the increase in the productivity of the highly educated worker in the high-tech sector results in higher education premium in both the high-tech and low-tech sectors. Using the historical U.S. data, calibration shows that SBTC can explain the general trends in the education premium and the residual wage inequality from 1963-2005. In particular, it solves the puzzle why the education premium fell but the residual wage inequality grew in the early 1970s.

*This is a part of my M.Phil. thesis under the guidance of Dr. Lu Chia-Hui, to whom I am indebted. Remaining errors are mine.
1 Introduction

This paper develops a variant of the standard search model to investigate the key factors behind the dynamics of education premium and residual wage inequality in the U.S. during the period between 1963 and 2005. Using the historical U.S. data to calibrate the model, we show that skill-biased technology change (SBTC) alone can explain the general trends of the wage inequality dynamics. In particular, our study reconciles four salient patterns of the wage inequality dynamics in the U.S. in the past four decades as documented by vast empirical studies. Specifically, the calibrated results show that: (i) the residual wage inequality exists not only in the highly educated group but also in the low-educated group, (ii) the residual wage inequality rises gradually, (iii) the education premium first falls in the 1970s, and then rises in the following period, and (iv) the education premium has grown at a faster speed than the residual wage inequality since the early 1980s.

We build an undirected search model with heterogeneous workers and firms. Workers differ in their education level and firms differ in the capital intensity of the job created. In contrast to the standard search model with deterministic, match-independent productivity of workers, our model features match-specific, stochastic productivity draw. The productivity of workers of different education level is realized and becomes observable only after the worker and firm match in the labor market. We show that the endogenous productivity thresholds for successful employment are sufficient statistics to pin down the equilibrium expected wage of each education-job pair matched. As firms are competing for limited workers, and workers are competing for limited jobs, a change of one productivity threshold in response to a change of exogenous parameter will generate a spillover effect to the others as the values of outside options facing firms and workers will vary accordingly. It is shown in the paper that it is the general equilibrium effect which makes the impacts of SBTC propagate across different sectors and labor groups.

We argue that SBTC alone can explain the wage inequality dynamics should both the effect of biased productivity premium and the effect of capital intensity dispersion be considered. We show that the explaining power of the SBTC argument is crippled if either channel is omitted. A detailed discussion

\footnote{Interested readers are referred to Bound and Johnson (1992), Katz and Murphy (1992), Levy and Murnane (1992), Juhn et al. (1993), Card and DiNardo (2002), Lemieux (2006) and Autor et al. (2008).}
This section discusses the evidence on the linkage between the U.S. wage inequality and SBTC. Section 2 presents the model setup, followed by the steady state equilibrium analysis in section 3. Calibration results are in section 4. Section 5 concludes the key findings of this paper.

1.1 SBTC and The U.S. Wage Inequality

This subsection provides the linkages between the wage inequality dynamics and two effects of SBTC, namely biased productivity premium (BPP) effect and capital intensity dispersion (CID) effect. Also, the key existing literature will be reviewed in this section.

Figure 1 illustrates the dynamics of the wage inequality using March Current Population Surveys. As shown in figure 1, the college/high school wage gap and the residual 90/10 wage gap, controlling the measures of education, experience and gender, could be interpreted as education premium and residual wage inequality respectively. Several features of the wage inequality are demonstrated in figure 1: (i) residual wage inequality increased gradually, (ii) education premium declined in the early 1970s and rose in the other periods, and (iii) education premium grew more rapidly than residual wage inequality since the 1980s. In the 1990s, studies supported the fact that SBTC is the driving force behind the wage inequality growth. A substantial amount of studies (Bound and Johnson, 1992; Katz and Murphy, 1992; Levy and Murnane, 1992; Juhn et al., 1993) document the rise in the wage inequality in the U.S. labor market in the 1970s and the 1980s. The growth was first attributed to the increase in the college-high school wage premium in the mid-1970s and the 1980s. (Bound and Johnson, 1992; Katz and Murphy, 1992). However, the observed characteristics such as education, working experience, age and gender can only explain a third of the wage inequality growth. Meanwhile, Juhn et al. (1993), Dinardo et al. (1996) and Gottschalk (1997) believe that the change in the residual wage inequality can explain most of the overall wage inequality growth. In particular, Juhn et al. (1993) argue that the increase in the relative demand for skill caused the residual wage inequality to rise in the 1970s and the 1980s. Krueger (1993), Berman et al. (1994), Autor et al. (1998), Bartel and Sicherman (1999) and Allen (2001) argue that the skill-biased technology change (SBTC) is the key factor behind such

\footnote{Data are from Autor et al. (2008).}
Figure 1: The U.S. Wage Inequality dynamics
Nevertheless, challenges to the SBTC hypothesis have emerged since the late 1990s. For example, as pointed out by Lemieux (2006), if SBTC increases the demand for skill and thus widened the residual wage inequality in the past four decades, the growth of the education premium should be recognized as well. However, as documented in the existing literature (Juhn et al., 1993; Card and DiNardo, 2002; Lemieux, 2006; Autor et al., 2008), the U.S. education premium dropped in the early 1970s. This might not reconcile with the SBTC hypothesis.

The existing literatures are surprised that SBTC is incapable in explaining the decline in the U.S. education premium in the mid 1970s because they only consider SBTC as the improvement in the productivity of the highly educated in the high-tech sector, namely biased productivity premium (BPP) effect. Since the skill-biased technology complements the skill the highly educated possess, SBTC keeps improving the productivity of the highly educated in the high-tech sector, generating a biased productivity premium (BPP) effect and increasing education premium. Another effect of SBTC, namely capital intensity dispersion (CID) effect, plays an important role in determining both the residual wage inequality and the education premium but is always neglected in the wage inequality literature. This effect has largely increased the capital intensity dispersion between the high-tech and the low-tech sectors since the 1960s, inducing the residual wage inequality to rise and the education premium to fall (which will be shown later). Furthermore, our calibration results, using the historical U.S. data, indicate that the simulated education premium and residual wage inequality are highly correlated with the historical ones only if both the BPP effect and the CID effect are taken into account.

It can be easily deduced that the BPP effect increases the relative demand for the highly educated and thus their wages, improving the education premium. Acemoglu (1998, 1999, 2002, 2003) endogenizes the decision on technological progress, and shows that the increase in the relative supply of highly educated improves the marginal returns of R&D firms to advance the technology that complements the highly educated workers, inducing SBTC. Furthermore, Acemoglu (1998, 2002, 2003) shows that SBTC, in response to the sharp increase in the relative supply of the highly educated workers,

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increases the residual wage inequality as to why it followed an increasing trend since 1970s. In fact, SBTC should have also improved the education premium in the 1970s; however, Acemoglu (1998) argues that the relative supply of the highly educated increased so suddenly that SBTC progressed but did not respond fast enough. As a result, the effect of the increase in the relative supply of the highly educated first dominated the effect of SBTC as to why the education premium kept falling in the 1970s until SBTC fully responded to the relative supply of the highly educated in the 1980s. This explains why the education premium first declined and then rose again.

Acemoglu (1998) succeeds in explaining the U.S. wage inequality dynamics; nevertheless, the evidence that the increase of the relative supply of highly educated workers is so sudden that SBTC did not response fast enough to such increase is rather weak. Furthermore, it is well known and is shown in figure 1 of Juhn et al.’s (1993) paper that the real wage in the lower end of the wage distribution kept falling in the 1970s and the 1980s. (i.e. $0 > \triangle w_{Lb}$) However, the residual wage inequality $w_{Lg}/w_{Lb}$ rises in Acemoglu’s (1998) model because $\triangle w_{Lg} > \triangle w_{Lb} > 0$, which is inconsistent with the historical U.S. wage dynamics. On the contrary, the implication of our model is $\triangle w_{Lg} > 0 > \triangle w_{Lb}$ (which will be shown later). In addition, under the assumption that regardless of the ability the highly educated workers and the low-educated workers can only work in the high-tech sector and low-tech sector respectively, Acemoglu (1998) succeeds in predicting that the U.S. residual wage inequality and the education premium dynamics during 1970-1996. Nevertheless, to relax this assumption by allowing the high ability workers and the low ability workers to work in the good job and the bad job respectively, Acemoglu (1998) will predict the education premium declined gradually and the residual wage inequality first declined in the 1970s and then rose again in the 1980s, which are inconsistent with the historical U.S. wage inequality dynamics. In contrast to Acemoglu (1998), our model predicts the same trend of the wage inequality dynamics as the historical data without restricting workers of any type to work in any particular sector. In the steady state equilibrium, our model predicts that a fraction of the highly educated workers are willing to work in the bad jobs because they are compensated with wages above their outside option values. Similarly, so long as the realized productivity is high enough, a good job is willing to hire the

\[^4\text{w}_{Lb}\text{ and }\text{w}_{Lg}\text{ are denoted as the wages of the low-educated workers in the low-tech sector (Bad Job) and the high-tech sector (Good Job) respectively.} \]
low-educated workers because lower wages are required compared to hiring the highly educated workers.

This paper shows that the fall in education premium and the rise in residual wage inequality in the 1970s could be explained by the increase in the capital intensity dispersion. The intuition is that the pioneering technology is more expensive than ever, generating the capital intensity dispersion (CID) effect. In equilibrium, a higher flow profit is required to create a high-tech vacancy. As a result, workers, on average, receive a higher wage in the high-tech sector due to rent-sharing, widening the residual wage inequality. Using the historical U.S. data, calibration results (in section 4) show that CID effect increases residual wage inequality and reduce education premium.

A few works in the existing literature examine the linkage between the capital intensity dispersion (CID) effect of SBTC and the wage inequality. As shown in figure 2, the upper end of the distribution of the capital intensity (as measured by capital-labor ratio) rose sharply over time while it has not changed much in the rest of the distribution. Caselli (1999) documents that the difference in the capital-labor ratio between the 90th and the 10th percentile that has risen since the 1960s. Dunne et al. (2004) indicate that the wage dispersion is mostly attributed to the distribution of the computer investment while Autor et al. (2003) pinpoint that the spread of computerization induces an increasing demand for the skills the highly educated possess. In addition, a simple empirical study by Leonardi (2007) supports the fact that capital intensity dispersion and residual wage inequality are positively related. Apart from the existing literature, we plot the dynamics of the capital intensity dispersion and residual wage inequality during 1963-2005 in figure 2. We observe that both the capital intensity dispersion and the residual wage inequality are moving on a similar trend. This evidence suggests that the CID effect of SBTC increases the relative demand for skills, thereby widening the residual wage inequality.

In fact, it is much more challenging to explain and model the residual wage inequality in the theoretical framework. Similar to Acemoglu (1998, 1999, 2002, 2003), the existing literature attributes the residual wage inequality to the unobserved innate ability between workers. This unobserved ability increases the workers' capacity to generate products in Acemoglu’s (1998,

5Residual wage inequality is from Autor et al. (2008). Capital intensity dispersion is measured by the ratio of the 97.5th percentile to 2.5th percentile capital-labor ratio. Capital and the number of employees are from COMPUSTAT.
Figure 2: Evidence on Capital Intensity Dispersion Effect of SBTC
1999, 2002, 2003) work or to adapt to new technologies (Galor and Moav, 2000). According to the critique of Aghion et al. (2002), since the innate ability of a worker is fixed, these kinds of models cannot capture the wage volatility along the employment histories of the worker.

This paper builds an undirected search model to explain the four salient patterns of the wage dynamics in the U.S. during 1963-2005. In theoretical search models of Albrecht and Vroman (2002), Shi (2002), Wong (2003), Blazquez and Jansen (2008) and Dolado et al. (2009), the skill-biased technology favors the highly educated; hence, their productivity is higher in the high-tech sector. To analyze the BPP effect of SBTC, these models predict that the BPP effect of SBTC improves the education premium. However, according to the critique of Lemieux (2006), the SBTC should not have reduced the education premium but at the same time increased residual wage inequality in the 1970s. Therefore, the BPP effect of the SBTC alone fails to explain the education premium dynamics in the U.S. in the early 1970s.

In Shi’s (2002) model, he proves a unique equilibrium using a directed search model with heterogenous firms and workers, in which both residual wage inequality and education premium arise. In particular, he succeeds in showing that the SBTC leads to a growth in residual wage inequality and education premium, which is the case in the U.S. after the 1980s. Moreover, he shows that the education premium grows at a faster speed than residual wage inequality due to the BPP effect.

However, in Shi’s (2002), the BPP effect of SBTC fails to explain the fall in the education premium in the 1970s. In addition, Shi (2002) shows that it is optimal for the skilled worker to apply to the high-tech sector only. Firstly, as criticized by Wong (2003), the model of Shi (2002) proves the unique equilibrium where overeducation does not exist. Nevertheless, it is documented that both the overeducation and undereducation are persistent using data of Canada (Frenette, 2004), Germany (Daly et al., 2000) and the U.S. (Rubb, 2003). Secondly, the absence of the overeducation implies the skilled only work in the high-tech sector and are thus paid the same, inducing no residual wage inequality in the highly educated group. However, the wage inequality literature supports the fact that residual wage inequality exists in both the upper and lower end of the distribution of the residuals (Katz and

6This type of model matches several empirical regularities of creation and destruction of jobs over the real business cycle. Interested readers are referred to Merz (1995) and Cole and Rogerson (1999). Wong (2003) shows that Mortensen-Pissarides model fails to explain the U.S. wage inequality dynamics.
Murphy, 1992; Lemieux, 2006; Autor et al., 2008). These are not in line with the empirical evidence.

In the undirected search models of Albrecht and Vroman (2002), Blazquez and Jansen (2008) and Dolado et al. (2009), the low-educated are incapable of handling jobs in the high-tech sector. As a consequence, no undereducation appears in their models, generating no residual wage inequality amongst the low-educated.

To interpret SBTC as the technology that complements the skill of the highly educated in the high-tech sector (BPP effect), Wong (2003) calibrates a standard search model and concludes that the Mortensen-Pissarides model\(^7\) fails to explain the inequality growth in the U.S. Utilizing the historical U.S. data, she finds that the BPP effect of SBTC increases the education premium and reduces the residual wage inequality in the Mortensen-Pissarides model. Wong (2003) therefore concludes that the Mortensen-Pissarides model is incapable of explaining the wage inequality growth in the U.S. in the 1970s and the 1980s. She suspects such failure arises because of the nature of the undirected search model, in which firms cannot direct the workers’ application by posting different wages.

Leonardi (2007) develops an undirected search model to capture the CID effect of SBTC on residual wage inequality. He proves that the increase in the capital intensity dispersion causes the residual wage inequality to rise if the general equilibrium effect is ignored. Labors are homogenous in Leonardi’s (2007) model, generating no education premium. Hence, no issue on education premium can be examined in his model.

This paper develops an undirected search model to examine (i) the BPP effect, (ii) the CID effect, and (iii) the total SBTC effect on education premium and residual wage inequality. In contrast to the existing search models of heterogenous labors, the introduction of stochastic productivity realization ensures the existence of undereducation and overeducation in this model. This allows us to examine the impact of SBTC on the residual wage inequality in the upper and lower end of the distribution of the residual. Calibration examines three cases: (i) the BPP effect, (ii) the CID effect, and (iii) the total SBTC effect. We find that the simulated residual wage inequality and education premium are highly correlated with historical U.S. data only if both

\(^7\) She defines the Mortensen-Pissarides model as the one with four elements: (i) the search friction that generates unemployment, (ii) a matching technology that computes the flow of matches, (iii) a wage that maximizes the Nash product during the Nash bilateral bargaining process, and (iv) free entry that exhausts all the expected rent.
the BPP effect and CID effect are taken into account (as shown in section 4), suggesting that the SBTC hypothesis and the standard undirected search model are capable in explaining the wage inequality dynamics.

2 Model Setup

In this model, time is continuous. Workers and firms are risk neutral and have the same discounted rate $r$. Two types of worker exist in the economy, the highly educated $H$ and the low-educated $L$. The population is constant and normalized to be one, i.e. $L_H + L_L = 1$. Two types of vacancy exist, the low-tech vacancy $b$ and the high-tech vacancy $g$, each of which can only hire one worker. Firms are free to create vacancies. They first decide what type of vacancy to create. They need to purchase and install the job specific equipment $k_j$, where $k_b = k$ and $k_g = \sigma k$, $\sigma > 1$ before opening a vacancy.$^8$

Workers and firms do not know the productivity, which is match-specific, before they contact each other during the job search process. Let $\delta_{ij}$ be the productivity level of worker $i$ in vacancy $j$, where $i \in \{H, L\}$ and $j \in \{b, g\}$. The productivity of the matched worker follows a Pareto distribution $\delta_{ij} \sim \text{Pareto}(a, x_{ij})$, in which the Pareto index $a > 1$ and the productivity distribution has a full support over $[x_{ij}, \infty)$, where $x_{ij} \geq 1$. The cumulative distribution function is:

$$F_{ij}(\delta) = \begin{cases} 1 - \left(\frac{x_{ij}}{\delta}\right)^a, & \text{for } \delta \geq x_{ij}; \\ 0, & \text{for } \delta < x_{ij}. \end{cases}$$

The assumption of Pareto productivity distribution captures two properties. First, the higher the productivity, the lower will be its likelihood, i.e. $f'(\delta) < 0$, where $f(\delta)$ is the density function. Second, no worker generates infinite amounts of goods, i.e. $\lim_{\delta \to \infty} f(\delta) = 0$.

We assume that the high-tech equipment complements the skill of the highly educated workers while the education level does not matter in the low-tech vacancies. For simplicity, we assume that $x_H > x_L \geq 1$, where $x_H \equiv x_{Hg}$, and $x_L \equiv x_{Lg} = x_{Hb} = x_{Lb}$. The assumption implies: (i) regardless of the education level, a worker when matched with a low-tech vacancy draws productivity from the same distribution and (ii) the highly educated worker

$^8$Studies like Acemoglu (2001) assumes a fixed cost amount $k_b$ and $k_g$ are exogeneous while Leonardi (2007) endogenizes these fixed costs to analyze the capital-labor ratio decision of firms.
is more capable of operating high-tech equipment and thus able to produce more outputs than the low-educated worker in a high-tech job.\(^9\)

Workers and firms realize the productivity of the match when they come into contact. They agree to form the job match if both of them are satisfied with the bargaining wage. In case a worker \(i\) and a vacancy \(j\) form a job match, \(\delta_{ij}\) units of output \(Y_j\) are generated until a shock arrives at an exogenous constant Poisson rate \(\lambda > 0\). When this shock arrives, the worker becomes unemployed and the job is then unfilled. If either one rejects to form the job match for the instant, the job-seeker remains unemployed and the vacancy remains unfilled.

In contrast to standard search models of heterogenous agents\(^{10}\), this model does not assume homogenous productivity level of the matches; instead, it introduces a stochastic productivity realization. This assumption captures the match-specific component, in which the productivity of a vacancy largely varies with workers of similar characteristics.\(^{11}\) As a result, a portion of workers \(L\) possess higher productivity than worker \(H\) does, and vice versa.

In the economy, workers derive their utilities from the final goods \(Y\) only. The production function of the final goods is given by:

\[
Y = \left[ \alpha Y_b^\gamma + (1 - \alpha) Y_g^\gamma \right]^{\frac{1}{\gamma}}, \quad \alpha \in (0, 1)
\]  

where \(Y_b\) and \(Y_g\) are the aggregate production of the intermediate good \(b\) and the intermediate good \(g\) respectively, and \(\gamma < 1\). Two intermediate goods are sold in the competitive markets so that their prices are determined by the corresponding marginal returns to the production of \(Y\). \(P_b\) and \(P_g\) are denoted as the prices of \(Y_b\) and \(Y_g\) respectively, which are as follows:

\[
P_b = \alpha \left( \frac{Y}{Y_b} \right)^{1-\gamma}, \quad P_g = (1 - \alpha) \left( \frac{Y}{Y_g} \right)^{1-\gamma}
\]  

\(^9\)The highly educated and the low-educated are just-qualified in the high-tech sector and the low-tech sector respectively. The highly educated are overeducated in the low-tech sector because their qualifications exceed the requirements of the occupation. Similarly, the low-educated are undereducated in the high-tech sector because they lack the qualifications required.

\(^{10}\)See Acemoglu (2001), Albrecht and Vroman (2002), Shi (2002), Wong (2003), Blazquez and Jansen (2008), Dolado et al. (2009)

\(^{11}\)Jovanovic (1979) also introduces the stochastic job matchings to analyze worker turnover.
2.1 Matching Technology

Let \( u_i \) and \( v_j \) be the unemployment of a worker \( i \) and a vacancy \( j \) respectively. Hence, \( u = u_H + u_L \) and \( v = v_b + v_g \) are respectively the unemployment and the amount of vacancies in the economy. The matching technology \( M(u, v) \) is assumed to be differentiable and increasing in its arguments, concave, and constant returns to scale. The contact rate for a vacancy can be written as \( M(u, v)/v \). Define \( q(\theta) \equiv M(u, v)/v \) where \( \theta \equiv v/u \) is the market tightness. It is straightforward to show that the labor contact rate is \( M(u, v)/u = \theta q(\theta) \). Also, one can show that \( q(\cdot) \) is a differentiable decreasing function. Furthermore, we assume \( \lim_{\theta \to 0} q(\theta) = \infty \), \( \lim_{\theta \to 0} \theta q(\theta) = 0 \), \( \lim_{\theta \to \infty} q(\theta) = 0 \) and \( \lim_{\theta \to \infty} \theta q(\theta) = \infty \).

After a worker \( i \) and a vacancy \( j \) meet via the matching technology function, both parties realize the productivity \( \delta_{ij} \) of the match. Let \( R^i(j) \) and \( R^j(i) \) be the reservation productivity level, above which a worker \( i \) and a vacancy \( j \) are willing to contract respectively. When the realized productivity level of the match satisfies the reservation productivity level requirement \( R^i(j) \) and \( R^j(i) \), the contract is signed. Otherwise the job-seeker remains unemployed and the vacancy remains unfilled in the next instant.\(^{12}\)

2.2 Labor Market

The equilibrium will be characterized through a series of Bellman equations. Let \( J^E_{ij} \) and \( J^U_i \) be the discounted present value of being employed of a worker \( i \) in a vacancy \( j \) and the discounted present value of a worker \( i \) being unemployed respectively. Let \( w_{ij}(\delta_{ij}) \) be the wage of the worker \( i \) in the vacancy \( j \). \( J^E_{ij}(\delta_{ij}) \) is written as:

\[
r J^E_{ij}(\delta_{ij}) = w_{ij}(\delta_{ij}) + \lambda (J^U_i - J^E_{ij}(\delta_{ij}))
\]

A worker \( i \) receives a wage \( w_{ij} \) in a vacancy \( j \) and separates from it at a rate \( \lambda \) to become unemployed. Since \( \partial J^E_{ij}(\delta_{ij})/\partial \delta_{ij} \) is strictly positive (as shown later), there exists a unique reservation productivity level \( R^i(j) \) such that

\[
J^E_{ij}(R^i(j)) = J^U_i
\]

Intuitively, a worker \( i \) accepts the job offer if the discounted present value of working in a vacancy \( j \) is at least as high as the outside option value \( J^U_i \). The

\(^{12}\)Empirical evidence strongly supports this job-contact effect. Interested readers are referred to Barron (1975) and Pissarides (1986).
uniqueness of $R^i(j)$ implies that a worker $i$ is willing to work in a vacancy $j$ for all realized productivity levels exceeding $R^i(j)$. Let $\phi_v \equiv v_b/v$ be the fraction of the low-tech vacancies amongst all vacancies. Similarly, we can write $J^U_i$ as:

$$rJ^U_i = z + \theta q(\theta)\{\phi_v \int [\max\{J^E_{ib}(\delta_{ib}), J^U_i\} - J^U_i]dF_{ib}(\delta_{ib}) + (1 - \phi_v) \int [\max\{J^E_{ig}(\delta_{ig}), J^U_i\} - J^U_i]dF_{ig}(\delta_{ig})\}$$

(4)

where $z \leq x^L$ is the non-market income. Also, let $J^F_{ij}(\delta_{ij})$ be the discounted present value of a filled vacancy $j$ that is occupied by a worker $i$ and $J^V_j$ be the discounted present value of an unfilled vacancy $j$. $J^F_{ij}(\delta_{ij})$ can be written as:

$$rJ^F_{ij}(\delta_{ij}) = P_j \delta_{ij} - w_{ij}(\delta_{ij}) + \lambda(J^V_j - J^F_{ij}(\delta_{ij}))$$

(5)

A filled vacancy $j$ receives revenue $P_j \delta_{ij}$ and rewards the worker $i$ a wage $w_{ij}(\delta_{ij})$. Again, a filled vacancy $j$ also separates from the match at a rate $\lambda$ to become unfilled. Similar to $R^i(j)$, it can be found that there exists a unique $R^j(i)$ such that:

$$J^F_{ij}(R^j(i)) = J^V_j$$

A vacancy $j$ contract with a worker $i$ only if the discounted present value of being filled $J^F_{ij}(R^j(i))$ is at least as high as their outside option value $J^V_j$. Let $\phi_u \equiv u_L/u$ be the unemployment of the low-educated. As for the discounted present value of being a vacancy $j$, it can be written as follows:

$$rJ^V_j = q(\theta)[\phi_u(\int [\max\{J^E_{Lj}(\delta_{Lj}), J^V_j\} - J^V_j]dF_{Lj}(\delta_{Lj})) + (1 - \phi_u)\int [\max\{J^E_{Hj}(\delta_{Hj}), J^V_j\} - J^V_j]dF_{Hj}(\delta_{Hj})]\$$

(6)

Free entry ensures that vacancies are created until all rents are exhausted in the equilibrium; hence, the following hold in the steady state equilibrium:

$$J^V_j = k_j$$

(7)

Following the literature (Acemoglu, 2001; Albrecht and Vroman, 2002; Wong, 2003; Leonardi, 2007), wage is the one that maximizes the Nash product $(J^E_{ij}(\delta_{ij}) - J^U_i(\delta_{ij}))^\beta(J^F_{ij}(\delta_{ij}) - J^V_j(\delta_{ij}))^{1-\beta}$ to split the matching surplus.

13The unemployment insurance can be financed by a lump sum tax payment.
where $\beta \in (0, 1)$ is the bargaining power of workers. It can be found that a wage $w_{ij}(\delta_{ij})$ is the solution of the following equations:

\[
J_E^i(\delta_{ij}) - J_U^i = \beta(J_E^i(\delta_{ij}) - J_U^i + J_F^i(\delta_{ij}) - J_V^j) \tag{8}
\]

Intuitively, a worker $i$ shares a fraction of the total matching surplus. Using the equations (3), (5) and (8), the wage equations are as follows:

\[
w_{ij}(\delta_{ij}) = rJ_U^i + \beta(P_j\delta_{ij} - rJ_U^i - rk_j) \tag{9}
\]

Intuitively, a worker $i$ is compensated with their outside option values and a fraction of the matching surplus. Using equations (3), (5) and (9), one can easily verify that $\partial J_E^i(\delta_{ij})/\partial \delta_{ij} > 0$ and $\partial J_F^i(\delta_{ij})/\partial \delta_{ij} > 0$. Using the uniqueness and the definition of $R^i(j)$ and $R^j(i)$, it is straightforward to show from equations (8) that $R^i(j) = R^j(i)$. Denote the reservation productivity level as $\delta_{ij}^R \equiv R^i(j) = R^j(i)$. For all the realized productivity level $\delta_{ij} < \delta_{ij}^R$, both parties agree not to form the job match. We can write the following equation:

\[
J_E^i(\delta_{ij}^R) = J_U^i \quad J_F^i(\delta_{ij}^R) = J_V^j \tag{10}
\]

When contacted, a worker $i$ and a vacancy $j$ form a job match at the rate of $1 - F(\delta_{ij}^R)$. Write $\Upsilon_i(\phi_v)$ as $\phi_v(1 - F_{ib}(\delta_{ib}^R)) + (1 - \phi_v)(1 - F_{ig}(\delta_{ig}^R))$, $e_{ij}$ as the number of employed workers $i$ in the vacancy $j$, and $\phi_e \equiv (e_{Lb} + e_{Lg})/e$ as the fraction of the low-educated employment among all the employment $e$. In the steady state, following equations hold:

\[
\lambda(1 - u)(1 - \phi_e) = \theta q(\theta)\Upsilon_H(\phi_v)u(1 - \phi_u) \tag{11}
\]

\[
\lambda(1 - u)\phi_e = \theta q(\theta)\Upsilon_L(\phi_v)u\phi_u \tag{12}
\]

\[
L_H = (1 - u)(1 - \phi_e) + u(1 - \phi_u) \tag{13}
\]

Equations (11) and (12) imply that $u_H$ and $u_L$ are constant respectively; hence, equation (13) implies that $\phi_e$ is constant. If $\phi_e$, $\phi_u$, $\phi_v$ and $\delta_{ij}^R$, $\forall i, j$ remain unchanged, the equations (11) and (12) also ensure that all $e_{ij}$ are constant in the steady state equilibrium.
3 Steady State Equilibrium

A steady state equilibrium is defined as value functions $J_{ij}^E$, $J_{ij}^F$, $J_{ij}^U$ and $J_{ij}^Y$, prices of two goods $P_j$, wages $w_{ij}$, reservation productivity level $\delta_{ij}^R$, the proportion of low-educated employment $\phi_e$ and low-educated unemployment $\phi_u$, the fraction of low-tech vacancy $\phi_v$, unemployment $u$ and market tightness $\theta$ such that the equations (2)-(8), (10)-(13) are satisfied for all $i \in \{H, L\}$ and $j \in \{b, g\}$. Using the equations (11) and (12), the steady state unemployment of the worker $i$ are given by:

$$u_H = \frac{\lambda e_H}{\theta q(\theta) \Upsilon_H(\phi_v)}, \quad u_L = \frac{\lambda e_L}{\theta q(\theta) \Upsilon_L(\phi_v)} \quad (14)$$

From equation (14), the steady state fraction of low-educated employment is as follows:

$$\phi_e = \frac{\Upsilon_L(\phi_v) \phi_u}{\Upsilon_L(\phi_v) \phi_u + \Upsilon_H(\phi_v)(1 - \phi_u)} \in (0, 1) \quad (15)$$

Using equations (14) and (15) and the accounting identity $e = 1 - u$, steady state unemployment is obtained as follows:

$$u = \frac{\lambda}{\lambda + \theta q(\theta) [\Upsilon_L(\phi_v) \phi_u + \Upsilon_H(\phi_v)(1 - \phi_u)]} \in (0, 1) \quad (16)$$

Using equations (13), (15) and (16), the fraction of low-educated unemployed among the unemployed can be obtained as:

$$\phi_u = \frac{(1 - L_H) [\lambda + \theta q(\theta) \Upsilon_H(\phi_v)]}{\lambda + \theta q(\theta) [\Upsilon_H(\phi_v)(1 - L_H) + \Upsilon_L(\phi_v)L_H]} \in (0, 1) \quad (17)$$

The steady state production of the goods $Y_j = e_{Hj}\mathbf{E}(\delta_{Hj}|\delta_{Hj} \geq \delta_{Hj}^R) + e_{Lj}\mathbf{E}(\delta_{Lj}|\delta_{Lj} \geq \delta_{Lj}^R)$ are as follows:

$$Y_b = \frac{(1 - u)\phi_v}{\Upsilon_L(\phi_v) \Upsilon_H(\phi_v)} (\phi_e \mathbf{E}(\delta_{Lb}) \Upsilon_H(\phi_v) + (1 - \phi_e) \mathbf{E}(\delta_{Hb}) \Upsilon_L(\phi_v))$$

$$Y_g = \frac{(1 - u)(1 - \phi_v)}{\Upsilon_L(\phi_v) \Upsilon_H(\phi_v)} (\phi_e \mathbf{E}(\delta_{Lg}) \Upsilon_H(\phi_v) + (1 - \phi_e) \mathbf{E}(\delta_{Hg}) \Upsilon_L(\phi_v)) \quad (18)$$

where $\mathbf{E}(\cdot)$ is the expectation operator. $\mathbf{E}(\delta_{ij})$ is the average realized productivity when a worker $i$ and a vacancy $j$ meet; however, the match generates output only if the realized productivity exceeds the reservation level. Hence,
on average, the observed output level of a worker $i$ is $E(\delta_{ij} | \delta_{ij} \geq \delta_{ij}^R)$ in a vacancy $j$. Using the equations (2) and (18), the steady state prices of two goods can be written as:

$$P_b = \alpha \left( \frac{1}{\phi_v} \right)^{1-\gamma}, \quad P_g = (1 - \alpha) \left( \frac{1}{1 - \phi_v} \right)^{1-\gamma}$$

Unsurprisingly, the higher the proportion of the vacancy $b$, the lower will be the price $P_b$. Using the equations (3), (4) and $J_{ij}^R(\delta_{ij}^R) = J_U^i$, it is interesting to note that the reservation wages $w_{ij}^R$ equal the outside option value $rJ_U^i$ in the steady state equilibrium. Similarly, using the equations (3), (4), (5) and (7), we have:

$$P_j \delta_{ij}^R = rJ_U^i + rk_j$$

Intuitively, a contract is signed only if the realized revenue $P_j \delta_{ij}$ is high enough to cover reservation wages $w_{ij}^R = rJ_U^i$ and rental cost $rk_j$. Using the equations (4) and (20), it is straightforward to prove that $J_H^i > J_L^i$. In the high-tech sector, the highly educated generate higher realized productivity level than the low-educated on average, generating a higher outside option value of the highly educated. Using the equations (20), one can easily show that $\delta_{ij}^R_H > \delta_{ij}^R_L$. Compared to the low-educated, higher reservation productivity level is required for the highly educated to contract because of their higher reservation wage. Using the equations (3), (4) and (9), the outside option values can be rewritten as follows:

$$rJ_U^i = z + \theta q(\theta)[\phi_v \left( \int_{\delta_{ib}}^{\delta_{ib}} (P_b \delta_{ib} - rJ_U^i - rk) dF_{ib}(\delta_{ib}) \right)$$

$$+ (1 - \phi_v) \left( \int_{\delta_{ig}}^{\delta_{ig}} (P_g \delta_{ig} - rJ_U^i - \sigma rk) dF_{ig}(\delta_{ig}) \right)]$$

It is straightforward to show that the outside option values straightly decrease with $\phi_v$ and increasing with $\theta$. Intuitively, the rise in $\phi_v$ increases the fraction of the low-tech vacancy that generates lower returns, reducing the expected returns of the unemployed. Also, the rise in $\theta$ increases the tightness of the labor market, improving the chance of the workers to find a vacancy and thus the outside option value. Using the equations (5), (6), (7) and (9), we

\[^{14}\text{Proof is given in the appendix.}\]
obtain two equilibrium conditions:

\[
\frac{(r + \lambda)}{(1 - \beta)} r_k = q(\theta)[\phi_u(\int_{\delta_{Lb}}^{\infty} (P_b \delta_{Lb} - P_b \delta_{Rb})dF_{Lb}(\delta_{Lb})) + (1 - \phi_u)(\int_{\delta_{Hb}}^{\infty} (P_b \delta_{Hb} - P_b \delta_{Rb})dF_{Hb}(\delta_{Hb}))]
\]

(22)

\[
\frac{(r + \lambda)}{(1 - \beta)} \sigma r_k = q(\theta)[\phi_u(\int_{\delta_{Lg}}^{\infty} (P_g \delta_{Lg} - P_g \delta_{Rg})dF_{Lg}(\delta_{Lg})) + (1 - \phi_u)(\int_{\delta_{Hg}}^{\infty} (P_g \delta_{Hg} - P_g \delta_{Rg})dF_{Hg}(\delta_{Hg}))]
\]

(23)

Using the equations (22) and (23), it can be shown that \(P_g > P_b\) in the steady state equilibrium. When the \(\phi_v\) is approaching 0, \(P_b \to \infty\) from equations (19). To equate the value of both sides of the low-tech locus (22), it requires \(\theta \to \infty\) when \(\phi_v \to 0\). Also note that \(\theta\) is finite in the high-tech locus (23) when \(\phi_v \to 0\). Therefore, the low-tech locus is above the high-tech one in the \(\theta-\phi_v\) plane. A similar argument can be applied to the case \(1 - \phi_v \to 0\); hence, they must intersect in the range \(\phi_v \in (0, 1)\). Proposition (1) summarizes the result.

Proposition 1. The steady state equilibrium always exists such that the equations (2)-(8), (10)-(13) are satisfied. In the equilibrium, \(\phi_v \in (0, 1)\), \(P_g > P_b\) and \(\delta^R_{Hj} > \delta^R_{Lj}\).

Proposition 2. Having an identical output level, the following inequalities hold in the steady state:

1. For all \(\delta > \max\{\delta^R_{Hj}, \delta^R_{Lj}\}\), \(w_{Hj}(\delta) > w_{Lj}(\delta)\).

2. For all \(\delta > \max\{\delta^R_{ib}, \delta^R_{ig}\}\), \(w_{ib}(\delta) < w_{ig}(\delta)\) iff \(\delta > \frac{(\sigma-1)r_k}{\beta (P_g - P_b)}\).

Proof. See the appendix.

The first and the second implication of the proposition (2) compare wages across education groups and within groups respectively. The first one implies that with identical output level the highly educated receive higher wages than the low-educated because of their higher outside option value. The second

\(^{15}\)Proof is given in the appendix.
implication predicts that generating the same amount of output level $\delta_{ij}$, a worker $i$ receives a higher wage in the high-tech sector only if the output level $\delta_{ij}$ is sufficiently high such that the revenue share differentials $\beta (P^*_g \delta_{ig} - P^*_b \delta_{ib})$ exceed the rental differentials $(\sigma - 1) rk$.

In the existing literature (Albrecht and Vroman, 2002; Shi, 2002; Blazquez and Jansen, 2008), it is a general implication that the highly educated receive a higher wage in the high-tech sector than in the low-tech sector because the revenue differentials are assumed to be higher than the rental differentials. Hence, this model gives an insight on the condition under which workers $i$ indeed receive lower wages in the high-tech sector than in the low-tech even though they might do similar work and generate a similar output level.

A worker $i$ generates $\delta_{ij}$ unit of output in sector $j$ and receives a wage $w_{ij}(\delta_{ij})$ only if $\delta_{ij} \geq \delta_{ij}^R$ when matched. On average, the observed wage of a worker $i$ is $\mathbb{E}(w_{ij}(\delta_{ij})|\delta_{ij} \geq \delta_{ij}^R)$ in the sector $j$. Wage inequality is taken to be a log ratio of the average observed wages of two groups of workers. Education premium can be written as follows:

$$\ln \frac{\mathbb{E}(w_{Hj}(\delta)|\delta \geq \delta_{Hj}^R)}{\mathbb{E}(w_{Lj}(\delta)|\delta \geq \delta_{Lj}^R)} = \ln \frac{\beta P_j \int_{\delta_{Hj}^R}^{\infty} \delta_{Hj} dF_{Hj}(\delta_{Hj})}{1-F_{Hj}(\delta_{Hj}^R)} + r J_H^U = \ln \frac{[a-(1-\beta)]r J_H^U + \beta rk_j}{[a-(1-\beta)]r J_L^U + \beta rk_j} > 0 \quad (24)$$

Education premium arises from the differentials in the outside option values of two education groups. Similarly, residual wage inequality is:

$$\ln \frac{\mathbb{E}(w_{ig}(\delta)|\delta \geq \delta_{ig}^R)}{\mathbb{E}(w_{ib}(\delta)|\delta \geq \delta_{ib}^R)} = \ln \frac{[a-(1-\beta)]r J_i^U + \beta \sigma rk}{[a-(1-\beta)]r J_i^L + \beta rk} > 0 \quad (25)$$

The capital intensity dispersion $(\sigma > 1)$ creates residual wage inequality in two different education groups. Proposition (3) summarizes the results.

**Proposition 3.** The steady state equilibrium wage structure of the economy are as follows:

1. $\mathbb{E}(w_{Hj}(\delta_{Hj})|\delta_{Hj} \geq \delta_{Hj}^R) > \mathbb{E}(w_{Lj}(\delta_{Lj})|\delta_{Lj} \geq \delta_{Lj}^R)$

2. $\mathbb{E}(w_{ib}(\delta_{ib})|\delta_{ib} \geq \delta_{ib}^R) < \mathbb{E}(w_{ig}(\delta_{ig})|\delta_{ig} \geq \delta_{ig}^R)$
The first implication compares mean wages across education groups. This implication also reconciles with educational mismatch literature\(^\text{16}\), which supports the fact that, on average, the overeducated (undereducated) are paid at a higher (lower) wage than their co-workers. In this model, the highly educated are overeducated in the low-tech sector while the low-educated are undereducated in the high-tech sector. Hence, this proposition predicts that the overeducated (the highly educated), on average, receive higher wages than their co-workers (the low-educated) in the low-tech sector. In addition, it predicts that the undereducated (the low-educated) are, on average, paid at a lower wage than their co-workers (the highly educated) in the high-tech sector.

The second implication of the proposition (3) compares mean wages within education groups. It predicts that on average a worker \(i\) receives higher wages in the high-tech sector than in the low-tech sector, which is consistent with two empirical findings. First, it predicts that for a worker \(i\) with identical education background, vacancies, that pay higher wages on average, are more capital intensive, which is in accord with the empirical findings from Abowd et al. (1999). Second, it predicts that the overeducated (undereducated) are on average paid at a lower (higher) wage, which is in line with empirical evidence\(^\text{17}\). This proposition completes the story of the wage structure that is documented in the educational mismatch literature.

Another important feature related to the wage inequality and the educational mismatch is captured in the proposition (4), which states that the presence of overeducation and undereducation in the steady state equilibrium. With the Pareto productivity distribution, it is straightforward to show that so long as \(\delta_{ib}^R < \infty\) and \(\delta_{ig}^R < \infty\) overeducation and undereducation will coexist in the equilibrium.

\(r J_i^U\) and \(\delta_{ij}^R\) are finite in the steady state equilibrium \(\forall i, j\). Using the equations (19), simple algebra shows that \(\lim_{\phi_v \to 0} P_b = \infty\), \(\lim_{\phi_v \to 0} P_g < \infty\) and \(\lim_{\phi_v \to 0} \phi_v P_b = 0\). Therefore, \(r J_i^U\) is finite when \(\phi_v \to 0\) from the equations (21). Similar arguments can be applied to the case \(\phi_v \to 1\). Hence, \(r J_i^U\) is finite for all \(\phi_v \in (0, 1)\). In other words, \(r J_i^U\) is finite in the steady state equilibrium according to the proposition (1). Using the equations (20), in the case where \(\phi_v \to 0\), \(\delta_{ib}^R = 0\) and \(\delta_{ig}^R < \infty\). Intuitively, when the


\(^{17}\)See footnote 16.
fraction of the vacancy approaches zero in the low-tech sector, the price of $Y_b$ approaches infinity. Therefore, a worker $i$ will accept all the job offers from sector $j$ when $\phi_b \to 0$. Similar arguments and economic intuition can be applied to the case $\phi_v \to 1$. Hence, $r_j^U$ and $\delta_{ij}^R$ are positive and finite for all $\phi_v \in (0, 1)$. Proposition (4) concludes the result.

**Proposition 4.** Overeducation and undereducation co-exist in the steady state equilibrium.

The proposition (4) is about the existence of both overeducation and undereducation in the steady state equilibrium. Their existence is important to the wage inequality literature in that overeducation and undereducation generate the residual wage inequality in the highly educated groups and the low-educated groups respectively. The wage inequality literature supports the fact that residual wage inequality exists in both the upper and lower end of the distribution of the residuals (Katz and Murphy, 1992; Lemieux, 2006; Autor et al., 2008). Moreover, Lemieux (2006) and Autor et al. (2008) find that residual wage inequality grows substantially among college-educated workers. Hence, analyzing the residual wage inequality dynamics necessitates a model in which both overeducation and undereducation coexist in the equilibrium. The existing models neglect either overeducation or undereducation. In contrast to these models, the introduction of stochastic productivity realization ensures the incidence of all types of educational mismatches in the steady state equilibrium, which accords with the empirical evidence (Daly et al., 2000; Rubb, 2003; Frenette, 2004) on the persistence of overeducation and undereducation.

4 Model Simulation

In this section, calibration of the BPP effect, the CID effect and the total effect of SBTC are conducted separately using the historical U.S. data during 1963-2005. The first two calibration exercises aim to show that the wage inequality dynamics cannot be captured if either BPP effect or CID effect is omitted as to why Wong (2003) concludes that Mortensen-Pissarides model

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cannot explain the wage inequality dynamics. The last calibration result will give a clear picture that the total effect of SBTC is capable in explaining both the education premium and the residual wage inequality by capturing four major features of the wage inequality dynamics.

Exogenous parameters include \( a, \alpha, \gamma, \lambda, r, m, z, L, \beta, \xi, x_L, x_H, \sigma \) and \( k \). The measures of the capital-labor ratio \( k \), the capital intensity dispersion \( \sigma \) and the biased productivity premium are mainly from COMPUSTAT. In the model, \( k \) is the capital-labor ratio of low-tech vacancies. Capital, acquired from COMPUSTAT, is deflated using the deflator in real 2005 dollar from the Bureau of Economic Analysis. The deflated capital is then divided by the number of employees from COMPUSTAT to obtain the real capital intensity. \( k_{p,t} \) denotes the \( p \) percentile of the capital-labor ratio in year \( t \). We assume that SBTC has no impact on the capital-labor ratio in the low-tech sector. \( k \) is the average of the 2.5th percentile capital-labor ratio \( k_{2.5,t} \) over the period 1963-2005 as follows:

\[
k = \frac{1}{2005 - 1963} \sum_{t=1963}^{2005} k_{2.5,t}
\]

Capital intensity is measured by the capital-labor ratio. Hence, \( k = 3.58 \) is used as the capital intensity in the low-tech sector. \( \sigma_t \) measures the capital intensity dispersion between a high-tech vacancy and the low-tech vacancy. This paper takes the ratio of the 97.5th percentile capital intensity to the 2.5th percentile capital intensity as the measure of the capital intensity dispersion each year, i.e. \( \sigma_t = k_{97.5,t}/k_{2.5,t} \).

A mean productivity is computed using an AK model, i.e \( a \bar{x}_{j,t}/(a - 1) = A k_{j,t} \), where \( a \bar{x}_{j,t}/(a - 1) \) is the mean of a Pareto productivity distribution, \( A \) is a fixed production parameter and \( k_{j,t} \) is the average capital-labor ratio in the sector \( j \). Hence, it is easy to deduce that the lowest realized productivity level equals \( \bar{x}_{j,t} = A k_{j,t}(a - 1)/a \). To capture the biased productivity dispersion effect in \( \bar{x}_{H,L} \), the lowest realized productivity level \( \bar{x}_L \) is normalized to one. Then, \( \bar{x}_{H,L} = (a(a - 1)Ak_{g,t}/a(a - 1)Ak_{b,t}) \times \bar{x}_L = k_{g,t}/k_{b,t} \). \( k_{b,t} \) and \( k_{g,t} \) are the average of the capital intensity of the bottom 5th percentile and the

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19 The 1st percentile of the capital-labor ratio is discarded because a few capital-labor ratios are close to zero in the lower end of the distribution of the capital-labor percentile.

20 From figure 2, one can notice that the lower end of the distribution of the capital-labor percentile has not changed much in the past 4 decades.
top 5th percentile of the distribution respectively as follows:

\[ k_{b,t} = \frac{1}{5} \sum_{p=2}^{6} k_{p,t}, \quad k_{g,t} = \frac{1}{5} \sum_{p=95}^{99} k_{p,t} \]

To reflect the equal importance of both goods \( Y_b \) and \( Y_g \), we set \( \alpha = 0.5 \) and \( \gamma = 0.5 \). Due to the lack of evidence on the Pareto index \( a \), and the Pareto index is set at \( a = 2 \). The rest of the parameters follow the existing literature. To match the sample average for job destruction rate (Davis and Haltiwanger, 1992), we set the destruction rate at \( \lambda = 0.055 \). Following Wong (2003), we assume that the matching function forms as Cobb-Douglas function \( q(\theta) = m \times \theta^{-\xi} \), where \( \xi \in (0,1) \), and the discount rate is set at \( r = 0.04 \) to reflect an annual rate of 4\%, the scale parameter of the job-contact function \( m = 0.768 \), the unemployment benefit at \( z = 0.475 \), the fraction of the highly educated as \( L = 0.281 \) and workers’ surplus share at \( \beta = 0.5 \). Similarly, we follow Albrecht and Vroman (2002) to set the elasticity of the matching function at \( \xi = 0.5 \).

Two effects (the BPP effect and CID effect) and the total effect of SBTC are examined in the following section. Calibration results support the fact that SBTC alone is sufficient to explain the dynamics of the education premium and the residual wage inequality in the past four decades. In addition, we show that the explaining power of the SBTC argument on education premium (residual wage inequality) is crippled if the biased productivity premium effect (residual wage inequality effect) is omitted.

### 4.1 Biased Productivity Premium Effect

Figure 3 illustrates the simulated education premium and residual wage inequality under the biased productivity premium effect. The capital intensity dispersion \( \sigma \) is constant in this study and is measured by its average as follows:

\[ \sigma = \frac{1}{2005 - 1963} \sum_{t=1963}^{2005} \sigma_t \]

Not surprisingly, figure 3 illustrates that education premium follows an increasing trend under this effect, which is in line with Juhn et al. (1993), Wong (2003) and Autor et al. (2008). Intuitively, the BPP effect of SBTC improves the productivity of the highly educated in the high-tech sector,
thereby increasing the flow profit of the match and thus rewarding them with a higher wage. Hence, the BPP effect improves the education premium in the high-tech sector. In addition, such increase in the average wages of the highly educated improves the outside option value of the highly educated $rJ_H^U$. Although the BPP effect takes place in the high-tech sector, the increase in $rJ_H^U$ rewards them higher wages in the low-tech sector. As a result, the BPP effect of SBTC improves the education premium in both sectors. From table 1, the correlations between historical U.S. education premium and the simulated ones in two sectors are over 0.9, reflecting the high explaining power of the BPP effect on education premium.

The BPP effect of SBTC alone fails to explain residual wage inequality dynamics in the U.S. According to table 1, the correlation between the simulated residual wage inequality in the highly educated group and the historical one is $-0.9486$. Figure 3 also demonstrates the falling trend of the residual wage inequality in the highly educated group.

Residual wage inequality arises from the capital intensity dispersion $\sigma$, as shown in the equation (25). The increase in the outside option value of the highly educated $rJ_H^U$ undermines the role of the capital intensity in their expected wages. Hence, the BPP effect of SBTC reduces the residual wage inequality in the highly educated group. The rise in $rJ_H^U$ increases their reservation productivity level and thus their unemployment. This increase in the unemployment of the highly educated competes for vacancies with the low-educated unemployed, thereby reducing the chance for the low-educated unemployed to fill a vacancy and thus their outside option values $rJ_L^U$. Similarly, the fall in $rJ_H^U$ intensifies the role of the capital intensity in their expected wages as shown in the equation (25), increasing the residual wage inequality in the low-educated.

In Wong (2003), the model simulation predicts that the BPP effect reduces the residual wage inequality, which is measured by the average of the residual wage inequality in two education groups. In our model, the BPP effect largely reduces the residual wage inequality in the highly educated group and does not change much of the residual wage inequality in the low-educated group. Taking the average of the residual wage inequality in two different education groups, the BPP effect also reduces the simulated residual wage inequality in our model. Without considering the CID effect of SBTC, Wong (2003) concludes that the Mortensen-Pissarides model is incapable of explaining the wage inequality in the U.S. and suspects that such failure arises from the nature of the undirected search model, in which firms can-
Figure 3: Simulated Results: Biased Productivity Premium Effect of SBTC
not direct the job search by posting different wages. This subsection gives a clear explanation as to why the BPP effect cannot analyze the U.S. wage inequality dynamics using the undirected search model.

4.2 Capital Intensity Dispersion Effect

This section calibrates the model to examine the capital intensity dispersion effect on residual wage inequality and education premium. In this section, the biased productivity premium $x_H$ is constant and is measured by its average as follows:

$$x_H = \frac{1}{2005 - 1963} \sum_{t=1963}^{2005} x_{H,t}$$

In this model, residual wage inequality rises with capital intensity dispersion, as proven in proposition (??). Also, as shown in figure 2, the capital intensity dispersion and the historical residual wage inequality are moving on a similar trend. Hence, one could expect that the simulated residual wage inequality in two education groups are increasing as demonstrated in figure 4. Table 1 shows that the correlations between the historical residual wage inequality and the simulated ones are over 0.8. These high and positive correlations reflect the high explaining power of the CID effect on residual wage inequality. Intuitively, the rise in the capital instalment cost requires a high-tech firm to have a higher flow profit so as to create a new vacancy. As a result, the mean wages of the workers increase in the high-tech sector, widening the residual wage inequality in both the highly educated group and the low-educated group.

Education premium generates from the differentials in the outside option values of two education groups $r_J U_i$, as shown in the equation (24). The increase in the capital intensity dispersion $\sigma$ undermines the role of the $r_J U_i$ differentials in their expected wages. Hence, the CIP effect of SBTC reduces the education premium. Figure 4 demonstrates the falling trends of the education premium due to the CIP effect of SBTC. In addition, table 1 shows the negative correlation between the historical education premium and the simulated one, suggesting the incapability of the CIP effect in explaining the education premium.
Figure 4: Simulated Results: Capital Intensity Dispersion Effect of SBTC
4.3 Total SBTC Effect

Considering both the biased productivity premium effect and the capital intensity dispersion effect, the simulated results, as shown in figure 5 and 5, illustrate that education premium was on an increasing trend in both sectors during the period 1963-2005. In particular, one can observe that education premium falls in the mid-1970s. In addition, figure 5 shows the variation of the residual wage inequality in both the highly educated group and the low-educated group, in which the simulated residual wage inequality was increasing during 1963-2005. Figure 5 indexes the wage inequalities based on their values in the 1970s. Both the indexed education premium and residual wage inequality do not change much before the 1980s. One can easily notice the simulation result that the indexed education premium has grown faster than the indexed residual wage inequality after the mid-1980s. Table 1 also shows the positive correlations between the simulated and the historical data. We can therefore conclude that the total SBTC effect can explain the dynamics of both the U.S. education premium and residual wage inequality during 1963-2005.

Thus far, it is the first model calibration in the existing literature that solely utilizes SBTC to simulate the education premium and the residual wage inequality such that (i) the residual wage inequality rises gradually, (ii) the education premium first falls in the 1970s and then rises in the following period, (iii) the education premium has grown at faster speed than the residual wage inequality since the mid-1980s, and (iv) the residual wage inequality exists not only in the highly educated group but also in the low-educated group.

As documented in the existing literature, wages kept falling in the lower end of the wage distribution since 1970s. In Acemoglu (1998), the wages increase in all the wage groups. His model succeeds in predicting the increase in the residual wage inequality; however, such increase arises because of $\Delta w_{Lg} > \Delta w_{Lb} > 0$. In contrast to Acemoglu (1998), our implication of the increase in the residual wage inequality amongst the low-educated is $\Delta w_{Lg} > 0 > \Delta w_{Lb}$. As illustrated in the figure 5, in the low-tech sector the average wage of the low-educated kept declining after 1970s. As explained above, the BPP effects improves the outside option values of the highly educated and increases their unemployment. The highly educated job-seekers compete jobs with the low-educated, thereby reducing the chance of the low-educated to meet the vacancies and thus lowering their outside option values. As a result,
Figure 5: Simulated Results: Total SBTC Effect

Education Premium in the Low-Tech Sector, 1963 - 2005

Education Premium in the High-Tech Sector, 1963 - 2005
the average wages of the low-educated falls. More, the CID effect requests workers to generate a higher productivity value so as to sign the contract in the high-tech sector. This increases the reservation productivity threshold of workers in this sector, increasing the average wages in the high-tech sector. Meanwhile, the CID effect increases the unemployment, reducing the outside option values of the low-educated and their average wages in the low-tech sector.

5 Conclusion

This paper analyzes two SBTC effects, biased productivity premium effect and capital intensity dispersion effect. The biased productivity premium improves the average productivity of the highly educated in high-tech vacancies, inducing the education premium to rise. Meanwhile, more sophisticated technology requires high-tech firms to create new vacancies at a higher installment cost. High-tech firms therefore possess higher capital-labor ratios than before, increasing the returns to the highly educated in the high-tech sector.
Figure 6: Correlation: Historical Data and Simulated Result

<table>
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<th>Historical Data</th>
<th>Simulated Results</th>
<th>Education Premium (The High-Tech Vacancy)</th>
<th>Education Premium (The Low-Tech Vacancy)</th>
<th>Residual Wage Inequality (The High-Educated)</th>
<th>Residual Wage Inequality (The Low-Educated)</th>
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<td>Residual Wage Inequality</td>
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<td>Education Premium</td>
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<td>0.9149</td>
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<tr>
<td></td>
<td>Total</td>
<td></td>
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<td>Residual Wage Inequality</td>
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Table 1. Correlation Between The Simulated Result and The Historical Data in the U.S., 1963-2005
firms. As a result, the capital intensity dispersion effect causes the wages of the highly educated to be more dispersed amongst firms with different technologies, widening the residual inequality.

This paper contributes to the existing literature by explaining both the education premium and the residual wage inequality solely using SBTC, concluding that SBTC is sufficient to understand the U.S. wage inequality dynamics during 1963-2005. The calibration result explains the puzzle why the residual wage inequality exists amongst the highly educated as well as the low-educated, the residual wage inequality rose gradually during 1963-2005, the education premium fell in the 1970s and rose in the other periods and the education premium grew faster than the residual wage inequality after the early 1980s.

6 Proof

6.1 Proof of the Footnote 14

Proof. Assume \( r_{J_H}^U \leq r_{J_L}^U \). The equations (20) imply \( \delta_{R_H}^j \leq \delta_{R_L}^j \). However, the equations (4) with \( \delta_{R_H}^j \leq \delta_{R_L}^j \) imply \( r_{J_H}^U > r_{J_L}^U \).

6.2 Proof of the Footnote 15

Proof. Assume \( \int_{\delta_{Lb}^R}^{\infty} (P_b x - r_{J_L}^U - r_k) dF_{Lb}(x) < \int_{\delta_{Lg}^R}^{\infty} (P_g x - r_{J_L}^U - \sigma r_k) dF_{Lg}(x) \). \( \delta_{Lb}^R < \delta_{Lg}^R \) implies \( P_g > P_b \). Suppose \( \delta_{Lb}^R \geq \delta_{Lg}^R \). Since \( \sigma > 1 \), equations (20) imply \( P_g > P_b \).

Assume \( \int_{\delta_{Hb}^R}^{\infty} (P_b x - r_{J_H}^U - r_k) dF_{Lb}(x) \geq \int_{\delta_{Hg}^R}^{\infty} (P_g x - r_{J_H}^U - \sigma r_k) dF_{Lg}(x) \). Using the equations (22) and (23), \( \sigma > 1 \) implies \( \int_{\delta_{Hb}^R}^{\infty} (P_g x - r_{J_H}^U - \sigma r_k) dF_{Hg}(x) > \int_{\delta_{Hb}^R}^{\infty} (P_b x - r_{J_H}^U - r_k) dF_{Hb}(x) \). \( \delta_{Hb}^R < \delta_{Hg}^R \) implies \( P_g > P_b \). Using the equations (20), \( \delta_{Hg}^R \geq \delta_{Hb}^R \) implies \( P_g > P_b \).

6.3 Proof of the Proposition (2)

Proof. Using the wages equations (9), we acquire the wage differentials of the worker in the same industry,

\[
w_{Hj}^j(\delta) - w_{Lj}^j(\delta) = (1 - \beta)(r_{J_H}^U - r_{J_L}^U)
\]

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Since \( rJ^U_H > rJ^L_H \), we can conclude that \( w_{Hj}(\delta) > w_{Lj}(\delta), \forall \delta > x_H \). Using the wage equations (9), we acquire the residual inequality of the worker \( i \),

\[
w_{ig}(\delta) - w_{ib}(\delta) = \beta[\delta(P_g - P_b) - (\sigma - 1)rk]
\]

Hence, for all \( \delta > \max\{\delta^R_{ib}, \delta^R_{ig}\} \), \( w_{ib}(\delta) < w_{ig}(\delta) \) iff \( \delta > \frac{(\sigma-1)rk}{\beta(P_g - P_b)} \).

\[\square\]

References


