The Hausman Test for Correlated Effects in Panel Data Models under Misspecification

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Abstract

We investigate the performance under misspecification of the Durbin-Wu-Hausman test for correlated effects with panel data. Consideration of size leads to a general proposition for quadratic forms in normal variate which gives conditions for a class of test statistics, which are chi-square under correct specification, to be oversized under misspecification.

In the case of measurement error, the Hausman test is found to be a test of the difference in asymptotic biases of between and within group estimators. However, its ‘size’ is sensitive to the relative magnitude of the intra-group and inter-group variations of the covariates, and can be so large as to call into question the use of the statistic in this case. We discuss an implementation of the alternative robust formulation of the test. Power considerations are presented, using a matched employee-employer data set.

Keywords: models with panel data, Hausman test, minimum variance estimators, quadratic forms in normal variables, Monte Carlo simulations

JEL Classification: C23, C12, C16, C15
1 Introduction

The Hausman test is the standard procedure used in empirical panel data analysis in order to discriminate between the fixed effects and random effects model.¹ The general set up can be described as follows. Suppose that we have two estimators for a certain parameter $\theta$ of dimension $K \times 1$. One of them, $\hat{\vartheta}_r$, is robust, i.e. consistent under both the null hypothesis $H_0$ and the alternative $H_1$, the other, $\hat{\vartheta}_e$, is efficient and consistent under $H_0$ but inconsistent under $H_1$. The difference between the two is then used as the basis for testing. Hausman (1978) shows that, under appropriate assumptions, under $H_0$ the statistic $h$ based on $(\hat{\vartheta}_R - \hat{\vartheta}_E)$ has a limiting chi-squared distribution:

$$h = \left(\hat{\vartheta}_r - \hat{\vartheta}_e\right)' \left[Var\left(\hat{\vartheta}_r - \hat{\vartheta}_e\right)\right]^{-1} \left(\hat{\vartheta}_r - \hat{\vartheta}_e\right) \sim \chi^2_K.$$ 

If this statistic lies in the upper tail of the chi-square distribution we reject $H_0$. If the variance matrix is consistently estimated, the test will have power against any alternative under which $\hat{\vartheta}_r$ is robust and $\hat{\vartheta}_e$ is not. Holly (1982) discusses the power in the context of maximum likelihood.

Hausman also shows that, again under appropriate assumptions,

$$Var\left(\hat{\vartheta}_r - \hat{\vartheta}_e\right) = Var\left(\hat{\vartheta}_r\right) - Var\left(\hat{\vartheta}_e\right)$$

It is well known that the assumptions used are sufficient but not necessary, as discussed in Ruud (2000), or Wooldridge (1995), or Newey and McFadden (1994). Further, while it may be convenient to estimate $Var\left(\hat{\vartheta}_r - \hat{\vartheta}_e\right)$ using this result, one can argue that using

$$Var\left(\hat{\vartheta}_r - \hat{\vartheta}_e\right) = Var\left(\hat{\vartheta}_r\right) - 2Cov\left(\hat{\vartheta}_r, \hat{\vartheta}_e\right) + (Var\left(\hat{\vartheta}_e\right))$$

may be more robust, and the trade-off between robustness and power should be considered.

The plan of the paper is as follows. Section 2 presents a robust formulation of the Hausman test for correlated effects, which is based on the construction of an auxiliary regression. We explain and discuss to what extent the use of artificial regressions may allow us to construct tests based on the difference between two estimators in a panel data model without making strong assumptions about the disturbances. The motivation underlying the implementation of the robust test is that the size distortion of the standard Hausman test in cases of misspecification of the variance-covariance matrix of the disturbances may be serious. This is investigated in Section 3. We formulate

¹This approach is also used by Durbin (1954) and Wu (1973). For this reason tests based on the comparison of two sets of parameter estimates are also called Durbin-Wu-Hausman tests, or DWH. For simplicity of exposition we will refer to the Hausman (1978) set up.
a general proposition for positive-definite quadratic forms in normal variate which gives conditions for a class of test statistics, which are chi-square under correct specification, to be oversized under misspecification. We then investigate measurement error in panels.

Section 4 compares the power of the standard Hausman test and the robust formulation presented in Section 2 using a Monte Carlo experiment. Section 5 concludes.

2 A robust test by artificial regression

Consider the model (??). Defining the disturbance term

$$\varepsilon_t = \eta_t + \nu_t,$$

the variance-covariance matrix of the errors is

$$\Sigma_{(NT \times NT)} = I_N \otimes \Omega$$

where

$$\Omega = \begin{pmatrix} \sigma^2_{\eta} + \sigma^2 & \cdots & \sigma^2 \\ \vdots & \ddots & \vdots \\ \sigma^2 & \cdots & \sigma^2 + \sigma^2 \end{pmatrix} = \sigma^2 I_T + \sigma^2_{\eta} \iota \iota'$$ (1)

and $\iota$ is a column vector of $T$ ones.

The unobserved heterogeneity implies correlation over time for single units, but there is no correlation across units.

Hausman and Taylor (1981) propose three different specification tests for the null hypothesis of uncorrelated effects: one based on the difference between the Within Groups and the Balestra-Nerlove estimator, another on the difference between the Balestra-Nerlove and the Between Groups and a third on the difference between the Within Groups and the Between Groups. They show that the chi-square statistics for the three tests are numerically identical. We now analyze the Hausman statistic constructed on the difference between the Within Groups and the Balestra-Nerlove estimator, commonly used in empirical work.

Let

$$\lambda = \frac{\sigma^2}{\sigma^2 + T\sigma^2_{\eta}}.$$

We write the Balestra-Nerlove estimator (Balestra and Nerlove, 1966) as a function of the variables in levels

$$\hat{\beta}_{BN} = \left( X'QX + \lambda X'MX \right)^{-1} \left( X'Q + \lambda X'M \right) Y$$ (2)
where

\[ Q = I_N \otimes Q^+, \]
\[ Q^+ = I_T - \frac{1}{T} \bar{u} \bar{u}', \]
\[ M = I_N \otimes M^+, \]
\[ M^+ = \frac{1}{T} \bar{u} ' = I_T - Q^+, \]

\[ X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \quad X_i = \begin{bmatrix} x_{i1}' \\ x_{i2}' \\ \vdots \\ x_{iT}' \end{bmatrix}, \quad y_i = \begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT} \end{bmatrix}. \]

\( Q^+ \) is the matrix that transforms the data to deviations from the individual time mean, \( M^+ \) is the matrix that transforms the data to averages. Rearranging

\[ \hat{\beta}_{BN} = \left[ X' [\lambda I_{NT} + (1 - \lambda) Q] X \right]^{-1} X' [\lambda I_{NT} + (1 - \lambda) Q] Y. \quad (3) \]

The variance is

\[ Var(\hat{\beta}_{BN}) = \sigma^2 \left[ X' [\lambda I_{NT} + (1 - \lambda) Q] X \right]^{-1}. \quad (4) \]

Similarly, using the \( Q \) matrix defined in formula (2), we write also the \textit{Within Groups} estimator as a function of the initial variables in levels

\[ \hat{\beta}_{WG} = [X' X]^{-1} X' Y. \quad (5) \]

The variance is

\[ Var(\hat{\beta}_{WG}) = \sigma^2 [X' X]^{-1}. \quad (6) \]

Also,

\[ Cov(\hat{\beta}_{BN}, \hat{\beta}_{WG}) = Var(\hat{\beta}_{BN}). \]

This is symmetric, and thus equal to \( Cov(\hat{\beta}_{WG}, \hat{\beta}_{BN}) \). Therefore, we obtain

\[ Var(\hat{\beta}_{BN} - \hat{\beta}_{WG}) = Var(\hat{\beta}_{WG}) - Var(\hat{\beta}_{BN}). \]

The equality (??) holds for \( \lambda \) known or otherwise fixed.

As we said, the case of estimated \( \lambda \) can be treated by using the Hausman-Taylor result that an algebraically identical test statistic can be constructed using the difference between \( \hat{\beta}_{WG} \) and the \textit{Between Groups} estimator \( \hat{\beta}_{BG} \). We obtain

\[ \hat{\beta}_{BG} = \left( \hat{\beta}_{WG} - \hat{\beta}_{BG} \right)' \left[ Var(\hat{\beta}_{WG}) + Var(\hat{\beta}_{BG}) \right]^{-1} \left( \hat{\beta}_{WG} - \hat{\beta}_{BG} \right) \]
as the estimators have zero covariance when we define the Between Groups estimator as usual:

\[
\hat{\beta}_{BG} = \left( X' MX \right)^{-1} X' MY \\
Var(\hat{\beta}_{BG}) = \sigma^2 (1 + \theta T) \left[ X' MX \right]^{-1}, \theta = \frac{\sigma^2}{\sigma^2}.
\] (7)

In this form, we can see that estimating \( \sigma^2 \) and \( \lambda \) (or \( \sigma^2 \eta \)) affects only the variance matrix of the test statistic.

A robust version of the Hausman test can be based on the use of an artificial regression. We estimate directly the variance of the difference of the two estimators. Moreover, this provides an estimate for this variance that is consistent and robust to heteroscedasticity and/or serial correlation of arbitrary form in the within groups covariance matrix of the random disturbances. These estimators are obtained using White’s formulae (White, 1984). It will be made clear to what extent the application of White’s heteroscedasticity consistent estimators of covariance matrices in a panel data framework may also allow for the presence of dynamic effects within groups.

Different artificial regressions have been proposed in the panel data literature to test for the presence of random individual effects, such as a Gauss-Newton regression by Baltagi (1996) or that proposed by Ahn and Lo (1996). However, the assumption of initial spherical disturbances has not been relaxed. As shown by Baltagi (1997, 1998), under the assumption of spherical disturbances, the three approaches, i.e. the Hausman specification test, the Gauss-Newton regression and the regression proposed by Ahn and Lo, yield exactly the same test statistic. Arellano (1993) first noted in the same panel data framework that an auxiliary regression can also be used to obtain a generalized test for correlated effects which is robust to heteroscedasticity and correlation of arbitrary forms in the disturbances. Davidson and MacKinnon (1993) list at least five different uses for artificial regressions including the calculation of estimated covariances matrices. We will use this device to estimate directly the variance between the two estimators without using equality (??). Furthermore, the application of White’s formulae (White, 1984) in the panel data case will lead to heteroscedasticity and autocorrelation consistent estimators of such variance. Therefore, we can use an artificial regression to construct a test for the comparison of different estimators which is robust to deviations from the assumption of spherical disturbances. From now on we will call this technique the HR-test, for Hausman-Robust test.

Next we present the auxiliary regression that was proposed by Arellano (1993) to test for random versus fixed effects in a static panel data model.

Consider the general panel data model for individual \( i \)

\[
y_i = X_i \beta + v_i, \quad i = 1, ..., N.
\]

This system of \( T \) equations in levels can be transformed into \((T - 1)\) equations in
deviations and one in averages. We obtain
\[
\begin{align*}
\left\{ \begin{array}{l}
y_i^* = x_i^* \beta + \mu_i^* \\
y_i = \overline{x}_i \beta + \overline{\mu}_i
\end{array} \right. \quad (T - 1) \text{ equations} \\
y_i = \overline{x}_i \beta + \overline{\mu}_i \quad 1 \text{ equation}.
\end{align*}
\]

Estimating by OLS the \( N(T - 1) \) equations in orthogonal deviations from individual time-means we obtain the \textit{Within Groups} estimator, i.e. \( \hat{\beta}_{WG} \). Estimating by OLS the \( N \) average equations we obtain the \textit{Between Groups} estimator, i.e. \( \hat{\beta}_{BG} \).

Let
\[
\beta_{WG} = E\left( \hat{\beta}_{WG} \right)
\]
and
\[
\beta_{BG} = E\left( \hat{\beta}_{BG} \right).
\]

Rewrite the system as
\[
\begin{align*}
\left\{ \begin{array}{l}
y_i^* = x_i^* \beta_{WG} + \mu_i^* - x_i^* \beta_{BG} + x_i^* \beta_{BG} \\
y_i = \overline{x}_i \beta_{BG} + \overline{\mu}_i
\end{array} \right.
\end{align*}
\]

Rearranging, we obtain
\[
\begin{align*}
\left\{ \begin{array}{l}
\overline{y}_i = x_i^* (\beta_{WG} - \beta_{BG}) + x_i^* \beta_{BG} + \mu_i^* \\
\overline{y}_i = \overline{x}_i \beta_{BG} + \overline{\mu}_i.
\end{array} \right.
\end{align*}
\]

Call
\[
\begin{align*}
Y_i^+ &= \begin{pmatrix} y_i^* \\ \overline{y}_i \end{pmatrix}, \\
W_i^+ &= \begin{pmatrix} x_i^* \\ 0 \\ \overline{x}_i \end{pmatrix}, \\
\beta^+ &= \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \beta_{WG} - \beta_{BG} \\ \beta_{BG} \end{pmatrix}, \\
\mu_i^+ &= \begin{pmatrix} \mu_i^* \\ \overline{\mu}_i \end{pmatrix}.
\end{align*}
\]

The augmented auxiliary model is
\[
Y_i^+ = W_i^+ \beta^+ + \mu_i^+, \quad i = 1, ..., N. \quad (8)
\]

If we estimate \( \beta^+ \) by OLS, we obtain directly the variance of the difference of the two estimators in the upper left part of the variance-covariance matrix of \( \beta^+ \). If we then estimate this covariance matrix using the White’s formulae and we perform a Wald test on appropriate coefficients, we obtain a reliable \textit{HR-test} comparing the two estimators we are interested in, namely \( \hat{\beta}_{WG} \) and \( \hat{\beta}_{BG} \). As first noted by Arellano (1993), under the assumption of spherical disturbances a Wald test on appropriate coefficients in the auxiliary regressions is equivalent to the standard Hausman test. Appendix 4 of O’Brien and Patacchini (2003), henceforth O’B&P, provides an analytical derivation of the result below.
Let

\[ H^+ = \frac{1}{T} i', H = I_N \otimes H^+, H' H = \frac{1}{T} \]

\[ \tilde{\beta}_{BG} = [(HX)'(HX)]^{-1}(HX)'(HY) = (X'MX)^{-1}X'MY \]

\[ \tilde{\beta}_{WG} = [(QX)'(QX)]^{-1}(QX)'(QY) = (X'QX)^{-1}X'QY \]

Further, let \( G^+ \) be Arellano and Bover’s (1990) forward orthogonal deviations matrix, \((T - 1) \times T\), such that

\[ G^+ i = 0, G^+ G'^+ = I_{(T-1)}, G'^+ G^+ = Q^+ = I_T - \frac{1}{T} i i' \]

\[ G = I_N \otimes G^+, GG' = Q, GG' = I_N \otimes I_{(T-1)} = I_N(T-1) \]

\[ \tilde{\beta}_{WG} = [(GX)'(GX)]^{-1}(GX)'(GY) = (X'QX)^{-1}X'QY \]

If

\[ k = \sqrt{T/(1+T\theta)} \]

then after using the residuals from the Between and Within regressions to calculate a consistent estimator \( \phi \) of \( \phi \), and thus \( \hat{k} \) of \( k \), we can construct the Hausman test by carrying out the artificial regression of \( Y^* = \begin{bmatrix} \hat{k}H Y \\ \hat{k}H X \\ G Y \end{bmatrix} \) on \( X^* = \begin{bmatrix} \hat{k}H X \\ 0 \\ G X \end{bmatrix} \), and constructing a Wald test on the first \( K \) coefficients. Such a test we refer to as an \( HR \) (Hausman Robust) test.

In what follows, we will clarify to what extent an application of White’s formulae for estimators of covariances matrices (White, 1984) in a panel data context provides a consistent estimator which is robust to heteroscedasticity and arbitrary correlation in the covariance matrix of the random disturbances. It may also control for the presence of fixed effects. This latter possibility may be accommodated if we make further assumptions, i.e. cross-sectional heteroscedasticity which takes on a finite number of different values.

Consider a simple panel data framework without fixed effects

\[ y_{i1} = \beta x_{i1} + \varepsilon_{i1} \]
\[ y_{i2} = \beta x_{i2} + \varepsilon_{i2} \]
\[ \vdots \]
\[ y_{iT} = \beta x_{iT} + \varepsilon_{iT}, \quad i = 1, \ldots, N, \]

where

\[ E(\varepsilon_i, \varepsilon_i') = \begin{pmatrix} \sigma^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma^2 \end{pmatrix} = \sigma^2 I_T = \Sigma. \]
Assume that in the complete model
\[ \Omega_{(NT \times NT)} = I \otimes \Sigma = \begin{pmatrix} \Sigma & 0 & \ldots & 0 \\ 0 & \Sigma & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \Sigma \end{pmatrix}. \] (9)

Define
\[ X_i = \begin{pmatrix} x_{i1} \\ \vdots \\ x_{iT} \end{pmatrix}_{(T \times 1)} \quad y_i = \begin{pmatrix} y_{i1} \\ \vdots \\ y_{iT} \end{pmatrix}_{(T \times 1)} \quad \epsilon_i = \begin{pmatrix} \epsilon_{i1} \\ \vdots \\ \epsilon_{iT} \end{pmatrix}_{(T \times 1)} \]

and rewrite the model as
\[ y_i = (T \times 1) = X_i \beta + \epsilon_i, \quad i = 1, \ldots, N. \] (10)

This formulation allows us to consider panel data in the framework defined in White (1984). If we assume no cross-sectional correlation and \( N \to \infty \), all the hypotheses underlying the derivation of White’s results are satisfied. Hence, Proposition 7.2 in White (1984, p. 165) applies.

\[ \widehat{\Sigma} = N^{-1} \sum_{i=1}^{N} \widehat{\epsilon}_i \epsilon_i' \xrightarrow{p} \Sigma \] (11)

and
\[ \widehat{\Omega} = I \otimes \widehat{\Sigma} \xrightarrow{p} \Omega. \]

However, while with uni-dimensional data sets we obtain heteroscedasticity consistent estimators because \( \epsilon_i \) is a scalar, in the two dimensional case \( \epsilon_i \) is a vector and we obtain a consistent estimator of the whole matrix \( \Sigma \). Hence, by applying the result (11) in the panel data case we obtain a consistent estimator of the variance covariance matrix of the disturbances that also allows for the presence of dynamic effects within groups.

Therefore, the estimators of the variance of the OLS estimators of \( \beta \) in the panel data model (10) can be obtained by
\[ \widehat{Var}(\beta) = \left[ \sum_{i=1}^{N} \left(X_i'X_i\right) \right]^{-1} \sum_{i=1}^{N} X_i'\widehat{\Omega}X_i \left[ \sum_{i=1}^{N} \left(X_i'X_i\right) \right]^{-1}. \] (12)

As stated by Arellano (1993), they are heteroscedasticity and autocorrelation consistent. Such estimators are the ones used in the implementation of the HR-test. This case is referred in White (1984) as contemporaneous covariance estimation.
However, White (1984) also implements consistent estimators in another case that explicitly takes into consideration a grouping structure of the data. Consider again the panel data model (10). Replace assumption (9) by

$$\Omega_{(NT\times NT)} = \begin{pmatrix} \Sigma_1 & 0 & \ldots & 0 \\ 0 & \Sigma_2 & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \ldots & 0 & \Sigma_N \end{pmatrix}.$$  

In this context, in a slightly different notation from that used by White (1984, p.172-173), suitable for the panel data framework, we can obtain consistent estimators of the covariance matrix $\Omega$ using

$$\hat{\Omega} = \text{diag}(\hat{\Sigma}_1, \hat{\Sigma}_2, \ldots, \hat{\Sigma}_N)$$

where

$$\hat{\Sigma}_i = T^{-1}\hat{e}_i\hat{e}_i'.$$

In other words, a consistent estimator for the covariance matrix of group $i$ is constructed by averaging the group residuals over only the observations in group $i$. In the balanced panel data case, their number is constant between groups and equal to $T$. This estimator is not only robust to autocorrelation of arbitrary form within groups but it also allows for the possibility that individual error covariance matrices may differ according to observable characteristics (such as region, union, race, etc....).

### 3 The Size of the Test

In this section we investigate the size distortion which occurs in the use of the standard Hausman test when the basic assumptions (Lemma 2.1 in Hausman 1978) are not satisfied.

Consider the panel data model (??) presented in Section 1. The Hausman test investigates the presence of specification errors of the form $\text{Cov}(x_{it}, \eta_i) \neq 0$. The robust version proposed in Section 2 tests such orthogonality assumption between explanatory variables and disturbances in presence of other forms of misspecification. In particular we are interested in a possible misspecification in the variance-covariance matrix of the disturbances arising, for instance, from the presence of measurement errors in variables. This case may be the rule rather than the exception in applied studies.

We want to test the hypothesis

$$H_0 : \text{Cov}(x_{it}, \eta_i) = 0$$

against the alternative

$$H_1 : \text{Cov}(x_{it}, \eta_i) \neq 0.$$
when
\[ \text{Var}(\varepsilon_i | x) \neq \Omega_i. \] (14)

Hausman (1978) shows that under \( H_0 \) the test statistic
\[ h = q' \hat{V}(\hat{q})^{-1} \hat{q} \sim \chi^2_k \] (15)
where, \( \hat{V}(\hat{q}) \) is the asymptotic variance of \( q \), and \( k \) is the length of \( q \). The same test statistic is obtained if we consider the vector \( \hat{q} \) equal to
\[ \hat{q}_1 = (\hat{\beta}_{WG} - \hat{\beta}_{BN}), \]
or
\[ \hat{q}_2 = (\hat{\beta}_{BG} - \hat{\beta}_{BN}), \]
or
\[ \hat{q}_3 = (\hat{\beta}_{WG} - \hat{\beta}_{BG}). \]

As Hausman and Taylor (1981) pointed out they are all nonsingular transformations of one another. The estimate of the variance covariance matrix used in the three cases is
\[ \hat{V}(\hat{q}_1) = \hat{V}(\hat{\beta}_{WG}) - \hat{V}(\hat{\beta}_{BN}), \]
or
\[ \hat{V}(\hat{q}_2) = \hat{V}(\hat{\beta}_{BG}) - \hat{V}(\hat{\beta}_{BN}), \]
or
\[ \hat{V}(\hat{q}_3) = \hat{V}(\hat{\beta}_{WG}) + \hat{V}(\hat{\beta}_{BG}). \]

If we are in presence of misspecification of the form (14), none of the above expressions gives a consistent estimate of the variance-covariance matrix, even under \( H_0 \). The distribution of the test statistic under \( H_0 \) need to be investigated. The nominal size may be quite different from the observed one.

To investigate the size distortion under normality, we use the distributions of quadratic forms in normal random variables.\(^2\) In particular, we use the following Lemma.\(^3\)

**Lemma 1** (in Lemma 3.2 in Vuong, 1989). Let \( x \sim N_K(0,V) \), with rank \( V \leq K \), and let \( A \) be an \( K \times K \) symmetric matrix. Then the random variable \( x'Ax \) is distributed as a weighted sum of chi-squares with parameters \( (K, \gamma) \), where \( \gamma \) is the vector of eigenvalues of \( AV \).

This implies that \( x'Ax \sim \chi^2_r \), where \( r = \text{rank}(A) \), if and only if \( AV \) is idempotent (Muirhead, 1982, Theorem 1.4.5).

If \( A = V^{-1} \), i.e. in cases of no misspecification, \( AV \) is idempotent. The theorem is satisfied and result (15) holds. The test statistic gives correct significance levels.

If \( A \neq V^{-1} \) but \( AV \) is idempotent then \( \text{rank}(A) < K \) and/or \( \text{rank}(V) < K \) but still (15) holds. We omit this case for simplicity of exposition.

\(^2\)See, among others, Muirhead (1982, Ch. 1), Johnson and Kotz (1970, Ch. 29).

\(^3\)This Lemma holds also in the asymptotic case (using the Continuous Mapping Theorem, e.g. White, 1984, Lemma 4.27).
If \( A \neq V^{-1} \) and \( AV \) is not idempotent, implying that the eigenvalues of \( AV \) are not 0 or 1, the asymptotic distribution of the Hausman test under \( H_0 \) is a weighted sum of central chi-squares

\[
h \sim \sum_{i=1}^{K} d_i z_i^2
\]

where \( z_i^2 \sim \chi_1^2 \) and \( d_i \) are the eigenvalues of \( AV \). This implies that the significance levels of the standard Hausman test are not correct.

Consider first the limiting case where \( d_1 \to K, d_i \to 0, i = 2, \ldots, K \). Figure 1 illustrates numerically that

\[
Pr \left[ K \chi_1^2 > \chi_{K,\alpha}^2 \right],
\]

where \( \chi_{K,\alpha}^2 \) is the critical value for a test of size \( \alpha \) under the \( \chi_1^2 \) distribution. In this illustration \( \alpha \) is set equal to 0.05.

In general we distinguish two effects: a scale effect if \( \sum_{i=1}^{K} d_i \neq K \), which is predictable (e.g. if \( d_i = 2 \ \forall \ i, \ h \sim 2\chi_1^2 \)) and a dispersion effect if \( d_i \neq d_j \), even if \( \sum_{i=1}^{K} d_i = K \). We normalize the weights and we conjecture that the dispersion effect is maximized in the limit if we put all the weight on the largest eigenvalue, say the first one. Indeed, a thorough numerical investigation for \( K = 2 \) and 3 verifies that this is the case when testing with size less than 8\% for \( K = 2 \), and 14\% for \( K = 3 \).

Figure 1 illustrates this case, i.e. the tail area of a \( \chi_k^2 \) is compared with the maximum tail area of \( K \chi_1^2 \). The graph shows that the size distortion is an increasing function of \( K \). For instance, if \( K \) is equal to 14, an inappropriate use of the Hausman test will give a probability of rejecting a true hypothesis of exogeneity which is almost 4 time larger than the nominal size.

In certain simple contexts an expression for the eigenvalues of \( AV \) can be analytically derived. For instance, a common source of misspecification in the variance covariance matrix occurs when elements of the regressor matrix contain measurement errors.

Suppose the true model is

\[
y_{it} = z_{it}' \beta + \eta_i + v_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T \tag{16}
\]

where \( z_{it}' \) is a \( 1 \times K \) vector of theoretical variables, \( \eta_i \sim iid \left( 0, \sigma^2_\eta \right) \), \( v_{it} \sim iid \left( 0, \sigma^2 \right) \) uncorrelated with the columns of \( z_{it} \) and \( Cov(\eta_i, v_{it}) = 0 \). The observed variables are

\[
x_{it} = z_{it} + m_{it},
\]

where \( m_{it} \) is a vector of measurement errors uncorrelated with \( \eta_i \) and \( v_{it} \). The estimated model is

\[
y_{it} = x_{it}' \beta + \eta_i + v_{it} - \beta' m_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T. \tag{17}
\]
In the case of exact measurement, i.e. $m_{it} = 0$,
\[
\begin{align*}
Var(y_{it}) &= E(\eta_i + v_{it})^2 = \sigma^2_\eta + \sigma^2, \\
Cov(y_{it}, y_{it-s}) &= Cov(x'_it^\prime + \eta_i + v_{it}, x'_{it-s}^\prime + \eta_i + v_{it-s}) \\
&= \sigma^2_\eta \quad \forall s.
\end{align*}
\]

The variance-covariance matrix is matrix (1). It can be written as
\[
\Sigma = I_N \otimes \Omega_i,
\]
where
\[
\Omega_i = \sigma^2I_T + \sigma^2_\eta \mu' = \sigma^2[I_T + \vartheta_1 \mu'], 
\]
and
\[
\vartheta_1 = \frac{\sigma^2_\eta}{\sigma^2}.
\]

If we assume that $m_{it} \sim iid (0, \Sigma_M)$, we obtain
\[
\begin{align*}
Var(y_{it}) &= E(\eta_i + v_{it} - \beta m_{it})^2 = \sigma^2_\eta + \sigma^2 + \beta'\Sigma_M \beta, \\
Cov(y_{it}, y_{it-s}) &= Cov(x'_it^\prime + \eta_i + v_{it} - \beta m_{it}, x'_{it-s}^\prime + \eta_i + v_{it-s} - \beta' m_{it-s}) \\
&= \sigma^2_\eta \quad \forall s \neq 0.
\end{align*}
\]
So
\[\Omega_i = (\sigma^2 + \beta'\Sigma_M\beta) I_T + \sigma_n^2 u_i' = (\sigma^2 + \beta'\Sigma_M\beta) (I_T + \vartheta_2 u_i'),\]  
(19)
and
\[\vartheta_2 = \frac{\sigma_n^2}{\sigma^2 + \beta'\Sigma_M\beta}.
\]

Consider now the exogeneity test based, for instance, on the comparison between \(\hat{\beta}_{BG}\) and \(\hat{\beta}_{WG}\). In this case, the measurement errors render \(\hat{\beta}_{BG}\) and \(\hat{\beta}_{WG}\) inconsistent. If we assume that
\[p \lim(\hat{\beta}_{BG} - \beta) = p \lim(\hat{\beta}_{WG} - \beta) = [\Sigma_{ZQZ}/(T - 1) + \Sigma_M]^{-1} \Sigma_M\beta\]
O’B&P show in their Appendix 5 that if the rows \(M_i \sim NID(0, \Sigma_M)\)
\[
\sqrt{N}(\hat{\beta}_{WG} - \hat{\beta}_{BG}) \xrightarrow{D} N(0, [1/(T - 1)] [\Sigma_{ZQZ}/(T - 1) + \Sigma_M]^{-1} \times
\]

\[
[(\sigma^2 + \beta'\Sigma_M\beta)\Sigma_{ZQZ}/(T - 1) + \sigma^2 \Sigma_M + \{\Sigma_M\beta'\Sigma_M + (\beta'\Sigma_M\beta)\Sigma_M\}] \times
\]

\[
[\Sigma_{ZQZ}/(T - 1) + \Sigma_M]^{-1} + [\Sigma_{ZMZ} + \Sigma_M]^{-1} \times
\]

\[
[T\sigma_n^2\Sigma_{ZMZ} + (\sigma^2 + \beta'\Sigma_M\beta)\Sigma_{ZMZ} + \sigma_n^2 T\Sigma_M + \sigma^2 \Sigma_M + \{\Sigma_M\beta'\Sigma_M + (\beta'\Sigma_M\beta)\Sigma_M\}]\]

\[
[\Sigma_{ZMZ} + \Sigma_M]^{-1}.
\]

The Hausman test
\[h = (\hat{\beta}_{WG} - \hat{\beta}_{BG})' \left[\frac{\text{Var}(\hat{\beta}_{WG}) + \text{Var}(\hat{\beta}_{BG})}{N} \right]^{-1} \frac{\text{Var}(\hat{\beta}_{WG} - \hat{\beta}_{BG})}{N} \frac{\text{Var}(\hat{\beta}_{WG} - \hat{\beta}_{BG})}{N}
\]
will have the same asymptotic distribution as
\[h_a = \sqrt{N}(\hat{\beta}_{WG} - \hat{\beta}_{BG})'p \lim \left[\frac{\text{Var}(\hat{\beta}_{WG}) + \text{Var}(\hat{\beta}_{BG})}{N}\right]^{-1} \frac{\text{Var}(\hat{\beta}_{WG} - \hat{\beta}_{BG})}{N}
\]
and O’B&P also show in their Appendix 5 that
\[
N\text{Var}(\hat{\beta}_{BG}) \xrightarrow{p} \{\sigma^2 + \beta'\Sigma_M\beta - \beta'\Sigma_M \left[\frac{1}{(T - 1)}\Sigma_{ZQZ} + \Sigma_M\right]^{-1} \Sigma_M\beta\} \times
\]
[\Sigma_{ZQZ} + (T - 1)\Sigma_M]^{-1}
and
\[
N\text{Var}(\hat{\beta}_{WG}) \xrightarrow{p} \{T\sigma_n^2 + \sigma^2 + \beta'\Sigma_M\beta - \beta'\Sigma_M [\Sigma_{ZMZ} + \Sigma_M]^{-1} \Sigma_M\beta\} \times
\]
[\Sigma_{ZMZ} + \Sigma_M]^{-1}.
Thus in terms of the notation of Lemma 3, for the asymptotic distribution

\[
V = \left[ 1/(T - 1) \right] \left[ \Sigma_{ZQZ}/(T - 1) + \Sigma_M \right]^{-1} \times \\
\left[ (\sigma^2 + \beta' \Sigma_M \beta) \Sigma_{ZQZ}/(T - 1) + \sigma^2 \Sigma_M + \{ \Sigma_M \beta' \Sigma_M + (\beta' \Sigma_M \beta) \Sigma_M \} \right] \times \\
\left[ \Sigma_{ZQZ}/(T - 1) + \Sigma_M \right]^{-1} + [\Sigma_{ZMZ} + \Sigma_M]^{-1} \times \\
\left[ T \sigma_y^2 \Sigma_{ZMZ} + (\sigma^2 + \beta' \Sigma_M \beta) \Sigma_{ZMZ} + \sigma^2 \Sigma_M + \{ \Sigma_M \beta' \Sigma_M + (\beta' \Sigma_M \beta) \Sigma_M \} \right] \\
[\Sigma_{ZMZ} + \Sigma_M]^{-1}.
\]

and

\[
A = \left[ \{ \sigma^2 + \beta' \Sigma_M \beta - \beta' \Sigma_M \left( \frac{1}{T - 1} \Sigma_{ZQZ} + \Sigma_M \right)^{-1} \Sigma_M \beta \} \times [\Sigma_{ZQZ} + (T - 1) \Sigma_M]^{-1} \right]^{-1}
\]

Consider first the case when \( \beta = 0 \).

\[
V = \left[ 1/(T - 1) \right] \left[ \Sigma_{ZQZ}/(T - 1) + \Sigma_M \right]^{-1} \times \\
\left[ \sigma^2 \Sigma_{ZQZ}/(T - 1) + \sigma^2 \Sigma_M \right] \times [\Sigma_{ZQZ}/(T - 1) + \Sigma_M]^{-1} \\
+ [\Sigma_{ZMZ} + \Sigma_M]^{-1} \times [T \sigma_y^2 \Sigma_{ZMZ} + \sigma^2 \Sigma_{ZMZ} + \sigma^2 \Sigma_M + \sigma^2 \Sigma_M] [\Sigma_{ZMZ} + \Sigma_M]^{-1} \\
= \left[ 1/(T - 1) \right] \sigma^2 [\Sigma_{ZQZ}/(T - 1) + \Sigma_M]^{-1} \\
+ [\Sigma_{ZMZ} + \Sigma_M]^{-1} (T \sigma_y^2 + \sigma^2) [\Sigma_{ZMZ} + \Sigma_M] [\Sigma_{ZMZ} + \Sigma_M]^{-1} \\
= \left[ 1/(T - 1) \right] \sigma^2 [\Sigma_{ZQZ}/(T - 1) + \Sigma_M]^{-1} \\
+ (T \sigma_y^2 + \sigma^2) [\Sigma_{ZMZ} + \Sigma_M]^{-1}
\]

\[
A = \left[ \sigma^2 [\Sigma_{ZQZ} + (T - 1) \Sigma_M]^{-1} + \{ T \sigma_y^2 + \sigma^2 \} \times [\Sigma_{ZMZ} + \Sigma_M]^{-1} \right]^{-1}
\]

so \( AV = I \). As a check, when \( \Sigma_M = 0 \),

\[
V = \sigma^2 [1/(T - 1)] \left[ \Sigma_{ZQZ}/(T - 1) \right]^{-1} + [T \sigma_y^2 + \sigma^2] [\Sigma_{ZMZ}]^{-1}
\]

\[
A = \left[ \sigma^2 [\Sigma_{ZQZ}]^{-1} + \{ T \sigma_y^2 + \sigma^2 \} [\Sigma_{ZMZ}]^{-1} \right]^{-1}
\]

which can be compared with O’B&Ph Appendix 3.

Now let \( \Sigma_Q = \Sigma_{ZQZ}/(T - 1) \), \( \sigma^{*2} = \sigma^2 + \beta' \Sigma_M \beta \), \( c = \Sigma_M \beta \), \( c^{*2} = \sigma^{*2} + T \sigma_y^2 \), so

\[
V = \left[ 1/(T - 1) \right] \left[ \Sigma_Q + \Sigma_M \right]^{-1} \left[ \sigma^{*2} [\Sigma_Q + \Sigma_M] + cc' \right] [\Sigma_Q + \Sigma_M]^{-1} + \\
[\Sigma_{ZMZ} + \Sigma_M]^{-1} \left[ \sigma^{*2} [\Sigma_{ZMZ} + \Sigma_M] + cc' \right] [\Sigma_{ZMZ} + \Sigma_M]^{-1}
\]

\[
= \left[ 1/(T - 1) \right] \left[ \sigma^{*2} [\Sigma_Q + \Sigma_M]^{-1} + dd' \right] + [\sigma^{*2} [\Sigma_{ZMZ} + \Sigma_M]^{-1} + ee']
\]

where \( d = [\Sigma_Q + \Sigma_M]^{-1} c \), and \( e = [\Sigma_{ZMZ} + \Sigma_M]^{-1} c \). These are just the inconsistencies, which we are assuming equal.

\[
A = \left[ 1/(T - 1) \{ \sigma^{*2} - c' [\Sigma_Q + \Sigma_M]^{-1} c \} \times [\Sigma_Q + \Sigma_M]^{-1} \right]^{-1}
\]

\[
+ \{ \sigma^{*2} - c' [\Sigma_{ZMZ} + \Sigma_M]^{-1} c \} \times [\Sigma_{ZMZ} + \Sigma_M]^{-1}
\]
The simplest case to examine is when $\Sigma_Q = \Sigma_{ZM} \iff p \lim \widehat{\beta}_{WG} = p \lim \widehat{\beta}_{BG}$ for all $\beta$; let $\Sigma_QM = \Sigma_Q + \Sigma_M = \Sigma_{ZM} + \Sigma_M$. Noting $d = e$, we have

$$V = \sigma^2\Sigma_QM^{-1} + 2dd'$$

where

$$\sigma^2 = [1/(T-1)]\sigma^2 + \sigma'^2$$

$$= [T/(T-1)]\sigma^2 + T\sigma^2_{\eta}$$

$$A = [\sigma^{++2}\Sigma_QM^{-1}]^{-1}$$

and $A^2V$ has the same eigenvalues as

$$A^{1/2}V A^{1/2} = \frac{\sigma^2}{\sigma^{++2}} I + \frac{2}{\sigma^{++2}} \Sigma_{Q/2}^{1/2} dd' \Sigma_{Q/2}^{1/2}$$

and has $K - 1$ eigenvalues of

$$k = \sigma^2/\sigma^{++2}$$

and one of

$$k + (2/\sigma^{++2})d'\Sigma_QMd = k + (2/\sigma^{++2})c'\Sigma_QM^{-1}c$$

$$= k + (2/\sigma^{++2})\beta'\Sigma_M\Sigma_QM^{-1}\Sigma_M\beta.$$ 

Thus the size distortion depends on scalar quantities,

$$k = \frac{\sigma^2}{\sigma^{++2}} = \frac{1}{1 - k^*},$$

$$k^* = \frac{\sigma^2 - \sigma^{++2}}{\sigma^2} = \frac{\beta'\Sigma_M \Sigma_QM^{-1} \Sigma_M\beta}{[T/(T-1)]\{\sigma^2 + \beta'\Sigma_M\beta\} + T\sigma^2_{\eta}}$$

and the larger root is

$$\frac{\sigma^2}{\sigma^{++2}} + \frac{2}{\sigma^{++2}} k^* \sigma^2 = \frac{1}{1 - k^*} [1 + 2k^*].$$

$$\beta'\Sigma_M \Sigma_QM^{-1} \Sigma_M\beta = \beta'\Sigma_M^{1/2} [\Sigma_M^{1/2} (\Sigma_Q + \Sigma_M)^{-1} \Sigma_M^{1/2}]^{-1} \Sigma_M^{1/2} \beta$$

$$= \beta'\Sigma_M^{1/2} [\Sigma_M^{-1/2} \Sigma_Q \Sigma_M^{-1/2} + I]^{-1} \Sigma_M^{1/2} \beta$$
If we now write
\[ \gamma = \Sigma^{1/2} \beta \]
\( \gamma \) is the vector of parameters in the model
\[
y_i = \left[ Z_i + M_i \right] \Sigma^{-1/2} \Sigma^{1/2} \beta + \eta_i + \varepsilon_i = Z_i^* \gamma + M_i^* \gamma + \eta_i + \varepsilon_i
\]
where the rows of \( M_i \) are \( NID(0, I) \) and \( Z_i^* = Z_i \Sigma^{-1/2} \Rightarrow Z_i = Z_i^* \Sigma^{1/2} \Rightarrow Z_i^* M^* Z_i = \Sigma^{1/2} Z_i^* M^* + Z_i^* \Sigma^{1/2} \)

\[
k^* = \gamma' \left[ \Sigma^{-1/2} \Sigma_{ZMZ} \Sigma^{-1/2} + I \right]^{-1} \gamma / \sigma^2 = \gamma' \left[ \Sigma_{Z^*M^*} + I \right]^{-1} \gamma / \left[ T/(T - 1) \right] \{ \sigma^2 + \gamma' \gamma \} + T \sigma^2_\gamma
\]  \( (20) \)

The components of the variance of \( y_{i,t} \) are
\[ \text{Var}(y_{i,t}) = \gamma' \gamma + \sigma^2_\eta + \sigma^2 \]
so an interpretation of our result is that if one takes one component of the variance, \( \gamma' \gamma \), downweights it by the between sums of squares of the unobserved ‘true’ variables (in the model with standardised measurement errors), to produce \( \gamma' \left[ \Sigma_{Z^*M^*} + I \right]^{-1} \gamma \), then the ‘size’ distortion depends on \( k^* \), as in \( (20) \), and the asymptotic distribution of the Hausman test is not \( \chi^2(K) \), but a weighted sum of \( K \) \( \chi^2(1) \), \( K - 1 \) weights being \( 1/(1 - k^*) \), with one of \( 1 + 3k^*/(1 - k^*) \). It also follows that a lower bound to the distortion is provided by multiplying a \( \chi^2(k) \) by \( 1/(1 - k^*) \).

A number of qualifications are in order. This only occurs if the inconsistency of within and between estimators is equal, and, further, the within group sum of squares matrix, and between group sum of squares matrix, are equal:
\[ \Sigma_{ZMZ} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} Z_i M^* Z_i = \Sigma_Q = \frac{1}{T - 1} \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} Z_i Q^* Z_i. \]
The equality of \( p \lim (\hat{\beta}_{BG} - \beta) \) and \( p \lim (\hat{\beta}_{WG} - \beta) \) is required to ensure that the asymptotic ‘size’ is not 1. (Thus the Hausman test can be regarded as a (consistent) test of equality of these ‘inconsistencies’). The equality of \( \Sigma_{ZMZ} \) and \( \Sigma_Q \) simplifies the result and is an aid to interpretability. We also assume that the rows of \( M_i \), the measurement errors, are \( NID(0, \Sigma_M) \). Some assumption about fourth moments is required, and this appears the simplest.

We can plot the size distortion for assumed values of \( T, K, \gamma' \gamma, \gamma' \left[ \Sigma_{Z^*M^*} + I \right]^{-1} \gamma, \sigma^2_\eta \) and \( \sigma^2 \). If \( T = 5 \) or \( 10, 1 \leq K \leq 10, \gamma' \gamma = 1, \sigma^2_\eta = \sigma^2 = 0.1, \) and \( \gamma' \left[ \Sigma_{Z^*M^*} + I \right]^{-1} \gamma = 0.5 \), we have Figure 2, evaluated by Monte Carlo (1 million replications).

We can relax the assumption that \( \Sigma_Q = \Sigma_{ZMZ} \) by observing that \( V \) is of the form
\[ V = k_1 B + k_2 C + d^* d'' \]
and $A$ is of the form
\[ A = (k_3B + k_4C)^{-1} \]
where
\[
B = [\Sigma_Q + \Sigma_M]^{-1}, \quad C = [\Sigma_{ZMZ} + \Sigma_M]^{-1}
\]
\[
k_1 = \frac{1}{(T-1)}\sigma^2, \quad k_2 = \sigma^{**2}
\]
\[
d^* = \{1 + 1/(T-1)\}^{1/2}d = \{T/(T-1)\}^{1/2}d,
\]
\[
k_3 = 1/(T-1)\{\sigma^2 - c'B^{-1}c\}, < k_1
\]
\[
k_4 = \{\sigma^{**2} - c'C^{-1}c\}, < k_2
\]
and $B$ and $C$ are positive definite. We see that $A$ is “too small”, and the test will be oversized.

\[
V = B^{1/2}[k_1I + k_2B^{-1/2}CB^{-1/2} + B^{-1/2}d^*d''B^{-1/2}]B^{1/2}
\]
\[
A^{-1} = B^{1/2}[k_3I + k_4B^{-1/2}CB^{-1/2}]B^{1/2}
\]

Let
\[
D = B^{-1/2}CB^{-1/2} = P\Lambda P'
\]
where $P$ is orthogonal, $\Lambda$ diagonal, with as diagonal elements $\lambda_i$ the eigenvalues of $D$. Then

$$V = B^{1/2}P[k_1I + k_2\Lambda + P'B^{-1/2}\mathbf{d}'\mathbf{d}''B^{-1/2}P]P'B^{1/2}$$
$$A = [B^{1/2}P[k_3I + k_4\Lambda]P'B^{1/2}]^{-1} = B^{-1/2}P[k_3I + k_4\Lambda]^{-1}P'B^{-1/2}$$

and thus

$$AV = B^{-1/2}P[k_3I + k_4\Lambda]^{-1}[k_1I + k_2\Lambda + P'B^{-1/2}\mathbf{d}'\mathbf{d}''B^{-1/2}P]P'B^{1/2}$$
$$= B^{-1/2}P[\text{diag}((k_3 + k_4\lambda_i)^{-1})\{\text{diag}(k_1 + k_2\lambda_i)$$
$$+ P'B^{-1/2}\mathbf{d}'\mathbf{d}''B^{-1/2}P]\}]P'B^{1/2}$$

which has the same eigenvalues as

$$\{\text{diag}(k_1 + k_2\lambda_i)$$
$$+ \text{diag}((k_3 + k_4\lambda_i)^{-1}\{P'B^{-1/2}\mathbf{d}'\mathbf{d}''B^{-1/2}P}\}$$

The second matrix has rank 1, and the eigenvalues of the whole matrix are bounded between the smallest of $k_{0,i} = (k_1 + k_2\lambda_i)/(k_3 + k_4\lambda_i)$ and the largest of $k_{0,i} + \mathbf{d}''B^{-1}\mathbf{d}'/(k_3 + k_4\lambda_i)$. $\lambda_i$ are the eigenvalues of $D = B^{-1/2}CB^{-1/2}$, or of $B^{-1}C = [\Sigma Q + \Sigma M][\Sigma_{ZMZ} + \Sigma M]^{-1}$. $\mathbf{d} = [\Sigma Q + \Sigma M]^{-1}\mathbf{c} = [\Sigma_{ZMZ} + \Sigma M]^{-1}\mathbf{c} = B\mathbf{c} = C\mathbf{c}$

$$\mathbf{d}''B^{-1}\mathbf{d}' = \{T/(T - 1)\}\mathbf{c}'B\mathbf{c} = \{T/(T - 1)\}\beta'\Sigma M[\Sigma_{ZMZ} + \Sigma M]^{-1}\Sigma M\beta$$
$$= \{T/(T - 1)\}\gamma' [\Sigma_{Z^{*MZ^{*}}} + I]^{-1}\gamma$$

$$k_1 = [1/(T - 1)]\sigma^{*2}, \sigma^{*2} = \sigma^2 + \beta'\Sigma M\beta = \sigma^2 + \gamma'\gamma$$
$$k_2 = \sigma^{*2} = \sigma^2 + T\sigma^2_\eta$$
$$k_3 = 1/(T - 1)\{\sigma^{*2} - \mathbf{c}'B^{-1}\mathbf{c}\} = 1/(T - 1)\left[\sigma^2 + \gamma'\gamma - \gamma' [\Sigma_{Z^{*MZ^{*}}} + I]^{-1}\gamma\right] < k_1$$
$$k_4 = \{\sigma^{*2} - \mathbf{c}'C^{-1}\mathbf{c}\} = \sigma^2 + \gamma'\gamma + T\sigma^2_\eta - \gamma' [\Sigma_{Z^{*MZ^{*}}} + I]^{-1}\gamma < k_2$$

Thus

$$\sigma^{*2} = [1/(T - 1)]\sigma^{*2} + \sigma^{*2} = k_1 + k_2$$
$$\sigma^{*+2} = [1/(T - 1)]\{\sigma^{*2} - \mathbf{c}'\Sigma^{-1}_{QM}\mathbf{c}\} + \sigma^{*2} - \mathbf{c}'\Sigma^{-1}_{QM}\mathbf{c} = k_3 + k_4$$

$$k_{0,i} = \frac{k_1 + k_2\lambda_i}{k_3 + k_4\lambda_i} = \frac{k_1 + k_2 + k_2(\lambda_i - 1)}{k_3 + k_4 + k_4(\lambda_i - 1)}$$
$$= \frac{\sigma^{*2}[1 + k_2(\lambda_i - 1)/\sigma^{*2}]}{\sigma^{*+2}[1 + k_4(\lambda_i - 1)/\sigma^{*+2}]} = k_3[1 + k_2(\lambda_i - 1)/\sigma^{*2}]$$
$$= \frac{[1 + k_4(\lambda_i - 1)/\sigma^{*+2}]}{[1 + k_4(\lambda_i - 1)/\sigma^{*+2}]}$$
Thus comparing this case with the $B = C$ case, we are introducing more variability into the eigenvalues, which as we have seen, may well increase the ‘size’ of the test. (Thus the ‘size’ is sensitive to the relative magnitude of the intra-group and inter-group variations of the covariates, $\Sigma_{ZQZ}$ and $\Sigma_{ZMZ}$). Our conclusion is somewhat dispiriting: a significant Hausman statistic may arise from measurement error, as it is implicitly comparing the inconsistencies: but cannot be used to test if the inconsistencies are equal, as the ‘size’ may considerably exceed its nominal value, even when the inconsistencies are equal.

4 A Power Comparison

The possible serious size distortion of the standard Hausman test motivates the formulation of the $HR$-test. Using the White estimators for the variance-covariance matrix, the test is robust to the presence of common sources of misspecification of the variance-covariance matrix, i.e. to arbitrary patterns of within groups dependence. In other words, using the notation in Lemma 3, $AV$ is idempotent and the nominal size is equal to the observed one. We now use a simulation experiment to investigate the relative power of the Hausman test and the $HR$-test. We are interested in a quantitative assessment of the possible power loss that may incur in using a robust version of the test, in absence of misspecification.

The postulated data generation process is the following.

We consider the model

$$y = \alpha x + \beta z + u,$$

where $y$, $x$, $u$ and $z$ are $(NT \times 1)$. The null hypothesis of the Hausman test is

$$Cov(u, x) = 0$$

and

$$Cov(u, z) = 0.$$

We assume $z$ exogenous variable and we generate $x$ correlated with $u$, so that the null hypothesis above is not satisfied. We consider

$$x = \gamma w + \varepsilon,$$  \hspace{1cm} (21)

where $x$, $w$, $\varepsilon$ are $(NT \times 1)$, $w$ is an exogenous variable and $(u, \varepsilon)$ are drawn from a bivariate normal distribution with a specified correlation structure.

The values from the exogenous regressors and the range of values for the parameters comes from the empirical case of study analyzed in Patacchini (2002). Using UK data, the following model is estimated.

$$lfillv_{it} = c + \gamma lunfv_{it} + \pi lutot_{it} + e_{it}, \quad i = 1, ..., 275; \quad t = 1, ..., 63$$

where $lfillv$ is the logarithm of the number of filled vacancies, $lunfv$ is the number of unfilled vacancies (stock variable) and $lutot$ is the number of unemployed in the area $i$ at time $t$, both expressed in logs, $c$ is a constant term, $e$ indicates a disturbance term.
The estimates of $\gamma$ and $\pi$, 0.5 and 0.4, have been used in the simulation experiment for $\alpha$ and $\beta$ respectively. Also, the best prediction for unfilled vacancies ($lunfv$) is found to be

$$lunfv_{it} = 1.2 \ln otv_{it}, \quad i = 1,\ldots, 275; \quad t = 1,\ldots, 63,$$

where $lnotv$ is the logarithm of the number of monthly notified vacancies (flow variable). In our experiment design, the real values for $lutot$ and $lnotv$ have been used as exogenous variables, i.e. respectively $z$ and $w$. The endogenous variable $lunfv$, i.e. $x$, has been constructed according to the structure (21)

$$x = 1.2w + \varepsilon.$$

The equation estimated is

$$y = 0.5x + 0.4z + u,$$

where $(u, \varepsilon)$ are constructed as draws from a multivariate normal distribution with the specified correlation coefficient $\rho$ of $(0, 0.05, 0.10, \ldots, 0.95)$.

Six sample sizes, typically encountered in applied panel data studies are used. The experiment is repeated 5000 times for each sample size and level of correlation. Figures 3 to 5 contain the results of the simulation experiment. The power is expressed in percentage.

The tables displayed compare $H_{\text{pow}}$, the power of the Hausman statistic ($H$-test):

$$h = (\hat{\beta}_{WG} - \hat{\beta}_{BG})' \left( \widehat{V}_{WG} + \widehat{V}_{BG} \right)^{-1} (\hat{\beta}_{WG} - \hat{\beta}_{BG})$$

with $HR_{\text{pow}}$, the power of the robust Hausman statistic ($HR$-test) obtained using the auxiliary regression detailed in Section (2):

$$hr = (\hat{\beta}_{WG} - \hat{\beta}_{BG})' \left[ \text{Var} (\hat{\beta}_{WG} - \hat{\beta}_{BG}) \right]^{-1} (\hat{\beta}_{WG} - \hat{\beta}_{BG}),$$

with different sample sizes. Figures 6 to 11 illustrate the relative power functions. The significance level has been fixed at 5%. $\bar{\rho}$ is the estimated level of correlation between $x$ and $u$ conditioned upon $w$. For each level of $\rho$, $H_{\text{pow}}$ and $HR_{\text{pow}}$ indicate the percentage of times we reject a false hypothesis if we use respectively the $H$-test or the $HR$-test.

In Table 1, 2 and 3 the number of cross-sectional units is held fixed at 25 and the number of time periods is varied respectively between 4, 10 and 20. In Table 4, 5 and 6 the number of cross-sectional units is held fixed at 275 and the number of time periods is varied respectively between 4, 10 and 20. Table 1 to 4 show that the performance of the $HR$-test is comparable with the one of the $H$-test, even better for values of $\rho$ greater than 0.3. In larger samples (Table 5 and 6) the performance of the $H$-test is superior but the power loss of the $HR$-test is not serious. The $HR$-test gives a very high rejection frequency for the false hypothesis of absence of correlation.
## Table 1: N=25, T=4

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<th>( HR_{pow} )</th>
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## Table 2: N=25, T=10

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Figure 3: Simulation Results
Table 3: N=25, T=20

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Table 4: N=275, T=4

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Figure 4: Simulation Results
Table 5: \( N=275, T=10 \)

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Table 6: \( N=275, T=20 \)

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Figure 5: Simulation Results
between $x$ and $u$, starting from levels of correlation around 0.3 (86.5% and 87.4% respectively in Table 5 and 6) and it detects the endogeneity problem almost surely as soon as $\rho$ is higher than 0.4 (97.9% and 98.9% respectively in Table 5 and 6). Taking the results as a whole, the simulation experiment provides evidence that the performance of the HR-test in terms of power is satisfying in large samples and even better than the one given by the H-test in small samples.

In addition, it is worthwhile noting that a version of the Hausman test implemented in most econometric software, which is generally used in empirical studies, is the one based on the comparison between $\hat{\beta}_{WG}$ and $\hat{\beta}_{BN}$, i.e.

$$h = \left(\hat{\beta}_{WG} - \hat{\beta}_{BN}\right)' \left(\hat{V}_{WG} - \hat{V}_{BN}\right)^{-1} \left(\hat{\beta}_{WG} - \hat{\beta}_{BN}\right).$$

The problem related with this approach is that, in finite samples, the difference between the two estimated variance-covariance matrices of the parameters estimates (i.e. $\hat{V}_{WG} - \hat{V}_{BN}$) may not be positive definite. In this cases, the use of a code implementing a different Hausman statistic or the formulation of the Hausman test using an auxiliary regressions (e.g. the one proposed by Davidson and McKinnon (1993, p. 236), which is now already implemented in some statistical packages, or the (robust) one presented in this paper) are needed to obtain a test outcome.

5 Conclusions

Our recommendation is to use the Hausman test framework for the comparison of appropriate panel data estimators and to construct a version of the test robust to deviations from the classical errors assumption, as proposed in this paper. This test, the HR-test, gives correct significance levels in common cases of misspecification of the variance-covariance matrix and has a power comparable to the Hausman test when no evidence of misspecification is present. The power of the HR-test is even higher in small samples. It can be easily implemented using a standard econometric package.

The relationship between the eigenvalues corresponding to the quadratic form that is the Hausman test statistic, and the size of the test, needs to be established in generality. A proof is under construction.

References


Figure 6: Power function comparison when N=25, T=4

Power curves of the standard Hausman test (H-test) versus the one of the robust formulation presented in Section 2 (HR-test) plotted against corr for various sample sizes.
Figure 7: Power function comparison when N=25, T=10
Figure 8: Power function comparison when N=25, T=20
Figure 9: Power function comparison when N=275, T=4
Figure 10: Power function comparison when N=275, T=10
Figure 11: Power function comparison when N=275, T=20