A Simulation Test for Continuous-Time Models

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Abstract: In this article, we propose a simulation method to implement Hong and Li’s (2005) transition density based test for continuous-time models. The idea is to simulate a sequence of dynamic probability integral transforms, which is the key ingredient of Hong and Li’s (2005) test. The proposed procedure is generally applicable no matter whether or not the transition density of a continuous-time model has a closed form, and is simple and computationally inexpensive. A Monte Carlo study shows that the proposed simulation test has very similar sizes and powers to Hong and Li’s (2005) test using the closed form of the transition density (when available). Furthermore, the performance of the simulation test is robust to the choice of the number of simulation iterations and the number of discretization steps between adjacent observations.

Keywords: Continuous-time model, Dynamic probability integral transform, Generalized residuals, Monte Carlo integration, Simulation, Transition density.

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1 Introduction

Continuous-time models are widely used in finance to capture the dynamics of economic time series, such as interest rates, stock prices and foreign exchange rates. In contrast to the vast literature on the estimation of continuous-time models, there has been relatively little effort on specification analysis for continuous-time models. Model misspecification generally yields inconsistent model parameter estimators, which could lead to misleading conclusions on inference and hypothesis testing. Moreover, a misspecified model can yield large errors in derivatives pricing, hedging, and risk management. Therefore, it is important to develop reliable specification tests for continuous time models.

There has been an increasing interest in testing for continuous-time models. This includes Ait-Sahalia (1996), Gao and King (2004), Thompson (2002), Chen and Gao (2005, 2007), Corradi and Swanson (2005, 2006), Li and Tkacz (2006), Li (2007), and Bhardwaj, Corradi and Swanson (2007). Most existing tests are based on the stationary (i.e., marginal) density of the underlying process. The stationary density usually has a closed form and therefore related test statistics are convenient to compute. However, because the stationary density cannot capture the full dynamics of the underlying process, related tests have no power against misspecified models which have the same stationary density as the data generating process. To overcome this drawback, Hong and Li (2005) propose a specification test using the transition density (i.e., the conditional density given the past history), which can capture the full dynamics of the underlying process. The basic idea is to construct a sequence of dynamic probability integral transforms, which will be i.i.d. U[0,1] under correct model specification (Diebold, Gunther and Tay, 1998). Hong and Li then test the i.i.d. U[0,1] property by using a nonparametric density estimator. Because of the i.i.d. property of the probability integral transforms, the finite sample performance of Hong and Li’s (2005) is robust to persistent dependence in data, which is not enjoyed by the existing tests. Furthermore, Hong and Li’s (2005) is generally applicable to testing both continuous- and discrete-time models, no matter whether they are univariate or multivariate.

An important issue in implementing Hong and Li’s (2005) test is the calculation of the dynamic probability integral transforms. When the transition density has a closed form, the probability integral transforms can be calculated via numerical integration of the transition density. However, most continuous-time models have no closed form solution. One has to use Ait-Sahalia’s (2002a, 2002b) Hermite polynomial method or the simulation methods of Pedersen (1995), Elerian, Chib and Shephard (2001) and Brandt and Santa-Clara (2002) to approximate the transition density. These methods are computationally expensive, particularly when the sample size is sufficiently large.

In fact, to implement Hong and Li’s (2005) test, one needs the dynamic probability integral transforms (i.e., integrals of the transition density) rather than the transition density itself. Based on this observation, we propose a simple and convenient simulation method to implement Hong and Li’s (2005) test. The idea is to simulate dynamic probability integral transforms directly which does not require the knowledge of the closed form or the simulation of the transition density. The procedure is generally applicable to test various time series models because neither closed form solution nor accurate approximation for the transition density is needed. The simulation test is computationally inexpensive. Our Monte Carlo study shows that the simulation test has very similar size and power performances to Hong and Li’s (2005) test using the closed form solution of the transition density (when available). Furthermore, the procedure is robust to the choices of the number of simulation iterations and the number of discretization steps between adjacent observations. We note that
Pedersen (1994) first proposed a simulation method to approximate the probability integral transforms. He noted the potential usefulness of the probability integral transforms in diagnostic checking of a diffusion model, but did not propose any test procedure. Our work fills this gap.

In Section 2, we describe Hong and Li’s (2005) test. In Section 3, we propose a simulation method to implement Hong and Li’s (2005) test. In Section 4, we conduct a simulation study on its finite sample performance. Section 5 concludes.

2 Nonparametric Specification Test

Hong and Li’s (2005) test is generally applicable to both continuous- and discrete-time models, whether they are univariate or multivariate. For simplicity, we consider testing a univariate continuous-time model

\[ dX_t = \mu(X_t, \theta)dt + \sigma(X_t, \theta)dW_t + dJ_t(X_t, \theta), \]

where \( \mu(X_t, \theta) \), \( \sigma(X_t, \theta) \) and \( J_t(X_t, \theta) \) are the drift, diffusion and jump processes respectively, \( \theta \in \Theta \) is a finite-dimensional parameter vector, \( \Theta \subset \mathbb{R}^p \) is a compact parameter space, \( W_t \) is a standard Brownian motion. Throughout, we assume that the process \( X_t \) is time-homogenous and stationary with an unknown transition probability density. Given the specifications on \( \mu(X_t, \theta) \), \( \sigma(X_t, \theta) \) and \( J_t(X_t, \theta) \), the model in (1) fully characterizes a transition density \( p(x|X_{t-\Delta}, \theta) \) for the process \( \{X_t\} \), where \( \Delta > 0 \). We say that the continuous-time model in (1) is correctly specified if there exists some unknown parameter \( \theta_0 \in \Theta \) such that the model-implied transition density coincides with the true transition density of \( \{X_t\} \). In this case, the continuous-time model can capture the full dynamics of \( \{X_t\} \).

Hong and Li (2005) propose a test for model (1) using a discretely observed random sample \( \{X_{\tau \Delta}\}_{\tau=1}^n \). For notational simplicity, we assume \( \Delta = 1 \). To describe this test, we consider the following dynamic probability integral transform:

\[ Z_\tau(\theta) = \int_{-\infty}^{X_\tau} p(x|X_{\tau-1}, \theta)dx, \quad \tau = 1, \ldots, n. \]  

When the model in (1) is correctly specified in the sense that there exists some \( \theta_0 \in \Theta \) such that \( p(x|X_{\tau-1}, \theta_0) \) coincides with the transition density of \( \{X_t\} \), then the sequence \( \{Z_\tau(\theta_0)\} \) is i.i.d. \( U[0,1] \) (Diebold, Gunther and Tay 1998). The series \( \{Z_\tau(\theta)\} \) can be called the "generalized residuals" of the transition density model \( p(x|X_{\tau-1}, \theta) \). The i.i.d. \( U[0,1] \) property provides a basis for testing the model. If \( \{Z_\tau(\theta)\} \) is not i.i.d.\( U[0,1] \) for all \( \theta \in \Theta \), then the model in (1) is not correctly specified.

Hong and Li (2005) measure the distance between a model-implied transition density and the true transition density by comparing a kernel estimator \( \hat{g}_j(z_1, z_2) \) for the joint density of the pair \( \{Z_t(\theta_0), Z_{t-j}(\theta_0)\} \) with unity, the product of two \( U[0,1] \) densities, where \( j \) is a lag order. The kernel estimator of the joint density is, for any integer \( j > 0 \),

\[ \hat{g}_j(z_1, z_2) = (n-j)^{-1} \sum_{\tau=j+1}^n K_h(z_1, \hat{Z}_\tau)K_h(z_2, \hat{Z}_{\tau-j}), \]

where \( \hat{Z}_\tau = Z_\tau(\hat{\theta}) \), \( \hat{\theta} \) is any \( \sqrt{n} \)-consistent estimator for \( \theta_0 \), and \( K_h(z_1, \hat{Z}_\tau) \) is a boundary-modified kernel.

\[ \text{The modified kernel is used because the standard kernel density estimator produces biased estimates near the boundaries of} \]

\[ \text{the support.} \]
defined below. For \( x \in [0, 1] \), we define

\[
K_h(x, y) = \begin{cases} 
  h^{-1}k \left( \frac{x-y}{h} \right) / \int_{-x/h}^{1} k(u)du, & \text{if } x \in [0, h), \\
  h^{-1}k \left( \frac{y-x}{h} \right), & \text{if } x \in [h, 1 - h), \\
  h^{-1}k \left( \frac{x-y}{h} \right) / \int_{-1}^{(1-x)/h} k(u)du, & \text{if } x \in [1 - h, 1],
\end{cases}
\]

(4)

where the kernel \( k(\cdot) \) is a prespecified symmetric probability density, and \( h \equiv h(n) \) is a bandwidth such that \( h \to 0, nh \to \infty \) as \( n \to \infty \). One example of \( k(\cdot) \) is the quartic kernel \( k(u) = \frac{15}{40} (1 - u^2)(1|u| \leq 1) \) where \( 1(\cdot) \) is the indicator function. We will use this kernel in our simulation study. In practice, the choice of \( h \) is more important than the choice of \( k(\cdot) \). Like Scott (1992), we choose \( h = \tilde{S} Z n^{-\frac{1}{5}} \), where \( \tilde{S} Z \) is the sample standard deviation of \( \{ \tilde{Z}_i \}_{i=1}^{n} \). This simple bandwidth rule attains the optimal rate for bivariate density estimation.

Hong and Li’s (2005) test statistic is based on a properly standardized version of the quadratic form between \( \tilde{g}_j(z_1, z_2) \) and 1, the product of two \( U[0, 1] \) densities:

\[
\hat{Q}(j) \equiv [(n-j)h] \int_0^1 \int_0^1 [\tilde{g}_j(z_1, z_2) - 1]^2 dz_1 dz_2 - h A_h^0] / V_0^{1/2},
\]

(5)

where the nonstochastic centering and scale factors

\[
A_h^0 \equiv \left[ (h^{-1} - 2) \int_{-1}^{1} k^2(u)du + 2 \int_0^1 \int_{-1}^{1} k(u)dv db \right]^2 - 1,
\]

(6)

\[
V_0 \equiv 2 \left[ \int_{-1}^{1} \left( \int_{-1}^{1} k(u+v)k(v)dv \right)^2 du \right]^2,
\]

(7)

and \( k_b(\cdot) \equiv k(\cdot) / \int_{-1}^{1} k(v)dv \).

Under correct model specification, \( \hat{Q}(j) \) \( \rightarrow^d \) \( N(0, 1) \) for any fixed lag order \( j > 0 \) as \( n \to \infty \). The first lag \( j = 1 \) is often the most informative and important, but other lags may also reveal useful information on model misspecification. Moreover, \( \text{cov}[\hat{Q}(i), \hat{Q}(j)] \to^p 0 \) for \( i \neq j \) as \( n \to \infty \). This implies that \( \hat{Q}(i) \) and \( \hat{Q}(j) \) are asymptotically independent whenever \( i \neq j \). As a result, we can simultaneously use multiple statistics \( \{ \hat{Q}(j) \} \) with different lags to examine at which lag(s) the i.i.d. \( U[0, 1] \) property is violated. On the other hand, \( \hat{Q}(j) \) \( \to \infty \) in probability as \( n \to \infty \) whenever \( \{ Z_t(\theta_0), Z_{t-j}(\theta_0) \} \) are not independent or \( U[0, 1] \). This ensures that the proposed test has power against model misspecification. See Hong and Li (2005) for more discussion.

### 3 Simulation-based Nonparametric Specification Test

To implement Hong and Li’s (2005) test, we need to calculate the dynamic probability integral transform or generalized residual \( Z_t(\hat{\theta}) \) in (2). When the transition density of a continuous-time model has a closed form, \( Z_t(\hat{\theta}) \) can be calculated via numerical integration. Unfortunately, the transition densities of most continuous-time models have no closed form. In such scenarios, one could use various approximation methods, such as the Hermite polynomial method of Ait-Sahalia (2002a, 2002b), or the simulation method of Pedersen (1995), Elerian, Chib and Shephard (2001), and Brandt and Santa-Clara (2002) to first approximate the transition data due to asymmetric coverage of the data in the boundary regions. The denominators of \( K_h(x, y) \) for \( x \in [0, h) \cup (1 - h, 1] \) ensure that the kernel density estimator is asymptotically unbiased uniformly over the entire support \([0,1]\) (Hong and Li, 2005).
We now develop a simple yet generally applicable simulation method to compute the generalized residuals. The idea is to directly simulate dynamic probability integral transforms rather than the transition density. To avoid confusion, we denote the realizations of the random sample \( \{X_\tau\}_{\tau=1}^n \) by \( \{x_\tau\}_{\tau=1}^n \). Then a realization of the generalized residual \( Z_\tau(\theta) \) is
\[
Z_\tau(\theta) = \int_{-\infty}^{x_\tau} p(x \mid X_{\tau-1} = x_{\tau-1}, \theta) \, dx
\]  
\[
eq \int_{-\infty}^{\infty} 1(x \leq x_\tau) p(x \mid x_{\tau-1}, \theta) \, dx
\]  
\[
eq E_\theta[1(X_\tau \leq x_\tau) \mid X_{\tau-1} = x_{\tau-1}], \quad \tau = 1, \ldots, n,
\]
where \( E_\theta(\cdot \mid \cdot) \) denotes the conditional expectation given \( X_{\tau-1} = x_{\tau-1} \) under the model-implied transition density \( p(x \mid x_{\tau-1} = x_{\tau-1}, \theta) \).

Equation (11) suggests a simple approach to approximating \( z_\tau(\theta) \) by Monte Carlo integration. First, conditional on the observation \( X_{\tau-1} = x_{\tau-1} \) at time \( \tau - 1 \), we use the null continuous-time model in (1) to simulate a sample path for the process \( \{X_t\} \) between time \( \tau - 1 \) and \( \tau \), and obtain a simulated observation \( \tilde{X}_\tau \) at time \( \tau \). For this purpose, we should choose an adequate discretization scheme in order to mimic the dynamics of the continuous-time model in (1). The interval between \( \tau \) and \( \tau + 1 \) is divided into \( M \) subintervals, whose length is \( \delta = M^{-1} \). In practice, the Euler scheme and the Milstein scheme are widely used. For the Euler scheme, with a sufficiently small \( \delta \), \( \tilde{X}_{\tau-1+i\delta} \) can be assumed to follow a conditional normal distribution, given the previous \( \tilde{X}_{\tau-1+(i-1)\delta} \). In other words, a random draw \( \tilde{X}_{\tau-1+i\delta} \) is generated recursively under the following distribution:
\[
\tilde{X}_{\tau-1+i\delta} \mid \tilde{X}_{\tau-1+(i-1)\delta} \sim N(\mu(\tilde{X}_{\tau-1+(i-1)\delta}, \theta)\delta, \sigma^2(\tilde{X}_{\tau-1+(i-1)\delta}, \theta)\delta), \quad i = 1, \ldots, M,
\]
with \( \tilde{X}_{\tau-1} = x_{\tau-1} \).

Alternatively, we can use the Milstein scheme, which is known to provide a more accurate approximation than the Euler scheme. Suppose \( \varepsilon_{\tau-1+i\delta} \sim i.i.d. N(0,1) \) is drawn for each \( i = 1, \ldots, M \). Then \( \tilde{X}_{\tau-1+i\delta} \) is generated recursively via the following formula:
\[
\tilde{X}_{\tau-1+i\delta} = \tilde{X}_{\tau-1+(i-1)\delta} + \mu(\tilde{X}_{\tau-1+(i-1)\delta}, \theta)\delta + \sigma(\tilde{X}_{\tau-1+(i-1)\delta}, \theta)\sqrt{\delta} \varepsilon_{\tau-1+i\delta}
\]  
\[
+ \frac{1}{2} \sigma(\tilde{X}_{\tau-1+(i-1)\delta}, \theta)\sigma'(\tilde{X}_{\tau-1+(i-1)\delta}, \theta)(\varepsilon_{\tau-1+i\delta}^2 - \delta), \quad i = 1, \ldots, M,
\]
where \( \sigma'(\cdot, \cdot) \) is the partial derivative with respect to the first argument of the function.

For each interval from \( \tau - 1 \) to \( \tau \), a simulation path, \( \{\tilde{X}_{\tau-1+i\delta}\}_{i=1}^M \), is generated, and then an observation \( \tilde{X}_\tau \) is obtained. For each given \( \tau \), we do so \( S \) times, where \( S \) is a prespecified number of simulation iterations for Monte Carlo integration. After repeating the procedure \( S \) times, we can obtain a simulated independent random sample \( \{\tilde{X}_\tau^{(s)}\}_{\delta=1}^S \) for each \( \tau = 2, \ldots, n \), conditional on \( X_{\tau-1} = x_{\tau-1} \). It follows that we can approximate \( z_\tau(\theta) \) by the following sample average.
When $M \to \infty$ and $S \to \infty$, the sample average $\bar{Z}_r(\theta, S, M)$ converges to $z_r(\theta)$ by the uniform law of large numbers. In practice, because $\theta_0$ is unknown, we have to replace it with an estimator $\hat{\theta}$. This results in a sequence of simulated generalized residuals $\{\tilde{Z}_r(\hat{\theta}, S, M)\}_{r=1}^{n}$ based on the estimator $\hat{\theta}$. When the estimator $\hat{\theta}$ is $\sqrt{n}$-consistent for $\theta_0$ under correct model specification, simulated generalized residual $\tilde{Z}_r(\hat{\theta}, S, M)$ will converge to $z_r(\theta_0)$ in probability as $M \to \infty, S \to \infty$, and $n \to \infty$.

We summarize our simulation procedure to implement Hong and Li’s (2005) test:

- Estimate the continuous-time model in (1) using any method that yields a $\sqrt{n}$-consistent estimator $\hat{\theta}$;
- Compute the simulated generalized residuals $\{\tilde{Z}_r = \tilde{Z}_r(\hat{\theta}, S, M)\}_{r=1}^{n}$ for some prespecified choices of $S$ and $M$;
- Compute Hong and Li’s (2005) test statistic $\tilde{Q}(j)$ in Equation (5) using the simulated sequence of estimated generalized residuals $\{\tilde{Z}_r\}_{r=1}^{n}$. We use $\tilde{Q}(j)$ to denote the simulated version of Hong and Li’s (2005) test. If $\tilde{Q}(j) > C_\alpha$, the upper tailed $N(0,1)$ critical value at significance level $\alpha$, then we reject the null hypothesis that the model is correctly specified at level $\alpha$.

4 Finite Sample Performances

We now examine the finite sample performance of the simulated version $\tilde{Q}(j)$ of Hong and Li’s (2005) test via a Monte Carlo study. We are interested in how close the performances of the simulated version $\tilde{Q}(j)$ and the original version $\hat{Q}(j)$ of Hong and Li’s (2005) are in terms of size and power. Moreover, since the simulated version $\tilde{Q}(j)$ involves the choice of the number ($S$) of simulation iterations and the number ($M$) of discretization steps between neighboring observations, we will examine the sensitivity of the simulated version $\tilde{Q}(j)$ to the choices of $S, M$. For comparison, we adopt the same simulation design as Hong and Li (2005).

4.1 Size

To examine the sizes of the tests $\tilde{Q}(j)$ and $\hat{Q}(j)$, we consider a Vasicek (1977) model:

$$dX_t = \kappa(\alpha - X_t)dt + \sigma dW_t$$

where $\alpha$ is the long run mean and $\kappa$ is the speed of mean reversion. The smaller $\kappa$ is, the stronger the serial dependence in $\{X_t\}$, and consequently, the slower the convergence to the long run mean. Like Hong and Li (2005), we set $(\kappa, \alpha, \sigma^2) = (0.85837, 0.089102, 0.002185)$ and $(0.214592, 0.089102, 0.000546)$ respectively. This generates low and high persistent dependence in data, respectively. It allows us to examine the robustness of the tests to persistence of dependence in data.

For each parameterization, we simulate 1000 data sets of a random sample $\{X_t\}_{t=1}^{n}$, with $n = 1000$. For each data set, we estimate a Vasicek model with unknown parameter $\theta = (\kappa, \alpha, \sigma^2)'$ via the maximum likelihood estimation (MLE) method. Because the Vasicek model has a Gaussian closed-form transition density,
the computation of the original version $\tilde{Q}(j)$ of Hong and Li’s (2005) test is feasible. To compute the simulated version $\tilde{Q}(j)$, we have to choose the number $(S)$ of simulation iterations and the number $(M)$ of discretization steps between neighboring observations. To examine the robustness of size and power of the simulated version $\tilde{Q}(j)$, we consider various combinations from $S = 200, 500, 1000$ and $M = 1, 3, 5$. We consider the empirical rejection rates using the asymptotic critical values (1.28 and 1.65) at the 10 and 5% significance levels, respectively.

Figure 1 and 2 report the empirical sizes of the tests under the high and low persistent dependence cases, respectively. Each figure, given the number of simulations $(S)$, provides the empirical sizes of the simulated version $\tilde{Q}(j)$, $j = 1, \ldots, 20$, for different choices of $M$. Also, the sizes of the original version $\tilde{Q}(j)$ denoted by "original" are provided. The original $\tilde{Q}(j)$ statistics give reasonable sizes in both the high (Figure 1) and low (Figure 2) persistent dependence cases. In all the cases, the size of the simulated version $\tilde{Q}(j)$ is nearly same as that of $\tilde{Q}(j)$ for all the lag orders, whatever combinations of $S$ and $M$ is chosen. Like $\tilde{Q}(j)$, the performance of the simulation version $\tilde{Q}(j)$ is not affected by the degree of persistent dependence.

4.2 Power

To investigate the powers of $\tilde{Q}(j)$ and $\tilde{Q}(j)$, we use four data generating processes (DGPs) considered in Hong and Li (2005):

- **DGP 1.** The CIR (1985) Model:
  
  $\frac{dX_t}{\sigma} = \kappa(\alpha - X_t)dt + \sigma \sqrt{X_t}dW_t,$  
  \hspace{1cm} (13)

  where $(\kappa, \alpha, \sigma^2) = (0.89218, 0.090495, 0.032742)$.

- **DGP 2.** Ahn and Gao’s (1999) Inverse-Feller Model:
  
  $\frac{dX_t}{\sigma} = X_t[\kappa + (\alpha^2 - \kappa\alpha)X_t]dt + \sigma X_t^{3/2}dW_t,$  
  \hspace{1cm} (14)

  where $(\kappa, \alpha, \sigma^2) = (0.181, 15.157, 0.67421)$.

- **DGP 3.** CKLS (1992) Model:
  
  $\frac{dX_t}{\sigma} = \kappa(\alpha - X_t)dt + \sigma X_t^\rho dW_t,$  
  \hspace{1cm} (15)

  where $(\kappa, \alpha, \sigma^2, \rho) = (0.0972, 0.0808, 0.52186, 1.46)$.

- **DGP 4.** Ait-Sahalia’s (1996) Nonlinear Drift Model:
  
  $\frac{dX_t}{\sigma} = (\alpha_1 X_t^{-1} + \alpha_0 + \alpha_1 X_t + \alpha_2 X_t^2)dt + \sigma X_t^\rho dW_t,$  
  \hspace{1cm} (16)

  where $(\alpha_1, \alpha_0, \alpha_1, \alpha_2, \sigma^2, \rho) = (0.00107, -0.0517, 0.877, -4.604, 0.64754, 1.50)$.

For each of these four alternatives, we generate 500 realizations of a random sample $\{X_t\}_{t=1}^n$ with size $n = 1000$. For all DGPs 1–4, we simulate data via the Milstein scheme. To reduce the discretization bias, we
simulate five observations each day and sample the data at daily frequency. For each data set, we fit a Vasicek model via MLE.

Figure 3 reports the powers of both the simulated version $\tilde{Q}(j)$ and original version $\hat{Q}(j)$ of Hong and Li’s (2005) test, as a function of lag order $j$ from 1 to 20, at the 5% significance level using asymptotic critical values. Under each DGP, the powers of $\tilde{Q}(j)$ and $\hat{Q}(j)$ are very close for each lag order $j$, and each combination of the choices of $(S, M)$ in computing $\tilde{Q}(j)$. Even when the number of simulation iterations $S = 200$ (not reported here), the power of $\tilde{Q}(j)$ is very similar to the power of $\hat{Q}(j)$. Both tests have all-round good power against the four alternatives. There is no power loss using the simulated version $\tilde{Q}(j)$.

5 Conclusion

In this paper, we propose a convenient simulation method to implement Hong and Li’s (2005) test for continuous-time models using discretely sampled data. The idea is to simulate the dynamic probability integrals rather than the transition density. The former is an ingredient of Hong and Li’s (2005) test. The proposed simulation test is simple and computationally inexpensive, and is generally applicable to various time series models whether or not the transition density has a closed form. There is no need to approximate or simulate the transition density. Our Monte Carlo study shows that the proposed simulation test performs, in terms of both size and power, very similarly to the original version of Hong and Li’s (2005) test using the closed form solution of the transition density (when available), and the performance of the simulation test is robust to various choices of the number of simulation iterations and the number of discretization steps between adjacent observations.

6 References


Figure 1:

The finite sample size performance of the original version $\hat{Q}(j)$ and simulated version $\tilde{Q}(j)$ statistics for high level of persistent dependence.
Figure 2:
The finite sample size performance of the original version $\tilde{Q}(j)$ and simulated version $\tilde{Q}(j)$ statistics for low level of persistent dependence.
Figure 3:
The finite sample power performance of the original version $\tilde{Q}(j)$ and simulated version $\tilde{Q}(j)$ statistics for univariate diffusions.